

MATHEMATICS

Chapter 8: Introduction to Trigonometry



Introduction to Trigonometry

1. Meaning (Definition) of Trigonometry

The word trigonometry is derived from the Greek words 'tri' meaning three, 'gon' meaning sides and 'metron' meaning measure.

Trigonometry is the study of relationships between the sides and the angles of the triangle.

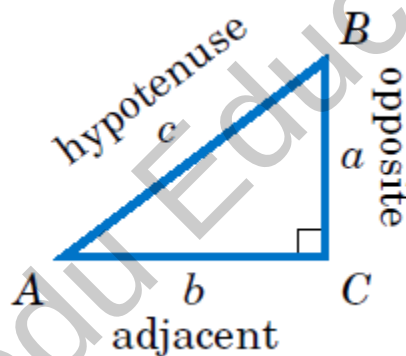
2. Positive and negative angles

Angle measured in anticlockwise direction is taken as positive angle whereas the angle measured in clockwise direction is taken as negative angle.

3. Trigonometric Ratios

Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios** of the angle.

Trigonometric ratios of the acute angle A in right triangle ABC are given as follows:



- i. $\sin \angle A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{p}{h}$
- ii. $\cos \angle A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{b}{h}$
- iii. $\tan \angle A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AC} = \frac{p}{b}$
- iv. $\operatorname{cosec} \angle A = \frac{\text{hypotenuse}}{\text{side opposite to } \angle A} = \frac{AB}{BC} = \frac{h}{p}$
- v. $\sec \angle A = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A} = \frac{AB}{AC} = \frac{h}{b}$
- vi. $\cot \angle A = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{AC}{BC} = \frac{b}{p}$

4. Important facts about Trigonometric ratios

- Trigonometric ratios of an acute angle in a right triangle represents the relation between the angle and the sides.
- The ratios defined above can be rewritten as $\sin A$, $\cos A$, $\tan A$, $\operatorname{cosec} A$, $\sec A$ and $\cot A$.

- Each trigonometric ratio is a real number and it has not unit.
- All the trigonometric symbols i.e., cosine, sine, tangent, cotangent, secant and cosecant, have no literal meaning.
- $(\sin\theta)^n$ is generally written as $\sin^n \theta$, n being a positive integer. Similarly, other trigonometric ratios can also be written.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

5. Pythagoras theorem:

It states that “in a right triangle, square of the hypotenuse is equal to the sum of the squares of the other two sides”.

Pythagoras theorem can be used to obtain the length of the side of a right angled triangle when the other two sides are already given.

6. Relation between trigonometric ratios:

The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

$$i. \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$ii. \quad \sec \theta = \frac{1}{\cos \theta}$$

$$iii. \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$iv. \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

7. Values of Trigonometric ratios of some specific angles:

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} A$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is

always greater than 1 or equal to 1.

- The value of $\sin\theta$ increases from 0 to 1 when θ increases from 0° to 90° .
- The value of $\cos\theta$ decreases from 1 to 0 when θ increases from 0° to 90° .
- If one of the sides and any other parts like either an acute angle or any side of a right triangle are known, the remaining sides and angles of the triangle can be obtained using trigonometric ratios.

8. Trigonometric ratios of complementary angles:

Two angles are said to be complementary angles if their sum is equal to 90° . Based on this relation, the trigonometric ratios of complementary angles are given as follows:

- $\sin(90^\circ - A) = \cos A$
- $\cos(90^\circ - A) = \sin A$
- $\tan(90^\circ - A) = \cot A$
- $\cot(90^\circ - A) = \tan A$
- $\sec(90^\circ - A) = \operatorname{cosec} A$
- $\operatorname{cosec}(90^\circ - A) = \sec A$

Note: $\tan 0^\circ = 0 = \cot 90^\circ$, $\sec 0^\circ = 1 = \operatorname{cosec} 90^\circ$, $\sec 90^\circ$, $\operatorname{cosec} 0^\circ$, $\tan 90^\circ$ and $\cot 0^\circ$ are not defined.

9. Definition of Trigonometric Identity

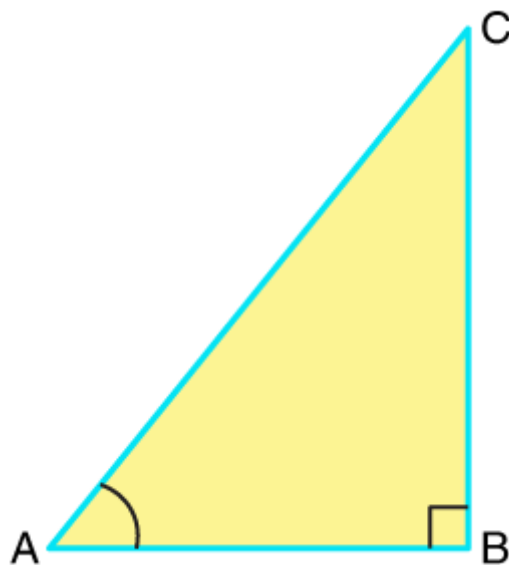
An equation involving trigonometric ratios of an angle, say θ , is termed as a **trigonometric identity** if it is satisfied by all values of θ .

10. Basic trigonometric identities

- $\sin^2 \theta + \cos^2 \theta = 1$
- $1 + \tan^2 \theta = \sec^2 \theta$; $0 \leq \theta < 90^\circ$
- $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$; $0 \leq \theta < 90^\circ$

11. Opposite & Adjacent Sides in a Right Angled Triangle

In the $\triangle ABC$ right-angled at B, BC is the side opposite to $\angle A$, AC is the hypotenuse and AB is the side adjacent to $\angle A$.



12. Trigonometric Ratios

For the right $\triangle ABC$, right-angled at $\angle B$, the trigonometric ratios of the $\angle A$ are as follows:

$$\sin A = \text{opposite side/hypotenuse} = BC/AC$$

$$\cos A = \text{adjacent side/hypotenuse} = AB/AC$$

$$\tan A = \text{opposite side/adjacent side} = BC/AB$$

$$\operatorname{cosec} A = \text{hypotenuse/opposite side} = AC/BC$$

$$\sec A = \text{hypotenuse/adjacent side} = AC/AB$$

$$\cot A = \text{adjacent side/opposite side} = AB/BC$$

13. Visualization of Trigonometric Ratios Using a Unit Circle

Draw a circle of the unit radius with the origin as the centre. Consider a line segment OP joining a point P on the circle to the centre which makes an angle θ with the x -axis. Draw a perpendicular from P to the x -axis to cut it at Q .

$$\sin \theta = PQ/OP = PQ/1 = PQ$$

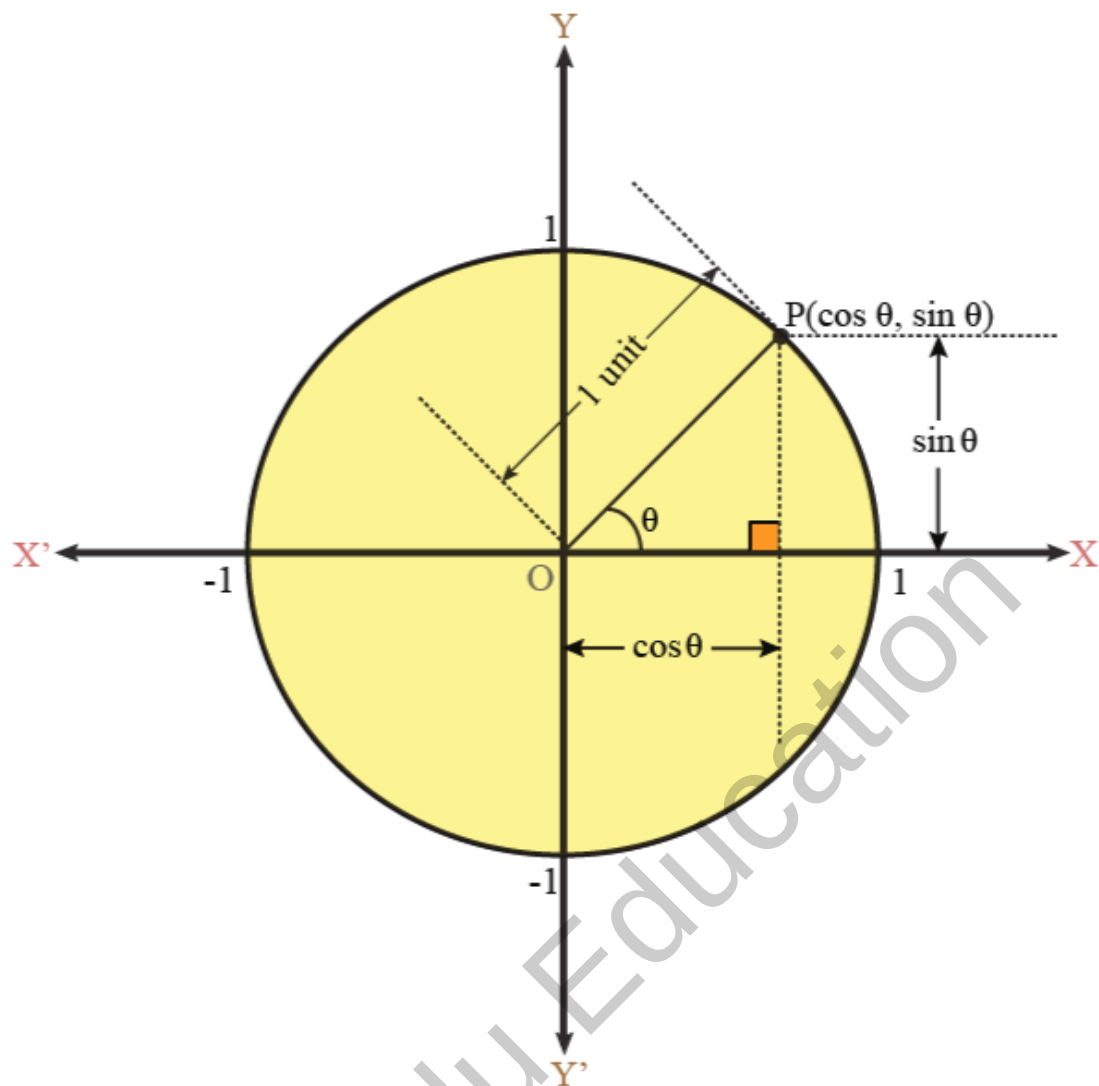
$$\cos \theta = OQ/OP = OQ/1 = OQ$$

$$\tan \theta = PQ/OQ = \sin \theta / \cos \theta$$

$$\operatorname{cosec} \theta = OP/PQ = 1/PQ$$

$$\sec \theta = OP/OQ = 1/OQ$$

$$\cot \theta = OQ/PQ = \cos \theta / \sin \theta$$



14. Relation between Trigonometric Ratios

$$\operatorname{cosec} \theta = 1/\sin \theta$$

$$\sec \theta = 1/\cos \theta$$

$$\tan \theta = \sin \theta / \cos \theta$$

$$\cot \theta = \cos \theta / \sin \theta = 1/\tan \theta$$

15. Range of Trigonometric Ratios from 0 to 90 degrees

For $0^\circ \leq \theta \leq 90^\circ$,

- $0 \leq \sin \theta \leq 1$
- $0 \leq \cos \theta \leq 1$
- $0 \leq \tan \theta < \infty$
- $1 \leq \sec \theta < \infty$
- $0 \leq \cot \theta < \infty$
- $1 \leq \operatorname{cosec} \theta < \infty$

$\tan \theta$ and $\sec \theta$ are not defined at 90° .

$\cot \theta$ and $\operatorname{cosec} \theta$ are not defined at 0° .

16. Variation of trigonometric ratios from 0 to 90 degrees

As θ increases from 0° to 90°

$\sin \theta$ increases from 0 to 1

$\cos \theta$ decreases from 1 to 0

$\tan \theta$ increases from 0 to ∞

$\operatorname{cosec} \theta$ decreases from ∞ to 1

$\sec \theta$ increases from 1 to ∞

$\cot \theta$ decreases from ∞ to 0

17. Standard values of Trigonometric ratios

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos A$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan A$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined
$\operatorname{cosec} A$	not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec A$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	not defined
$\cot A$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0

18. Complementary Trigonometric ratios

In Mathematics, the complementary angles are the set of two angles such that their sum is equal to 90° . For example, 30° and 60° are complementary to each other as their sum is equal to 90° . In this article, let us discuss in detail about the complementary angles and the trigonometric ratios of complementary angles with examples in a detailed way.

If θ is an acute angle, its complementary angle is $90^\circ - \theta$. The following relations hold true for trigonometric ratios of complementary angles.

$$\sin (90^\circ - \theta) = \cos \theta$$

$$\cos (90^\circ - \theta) = \sin \theta$$

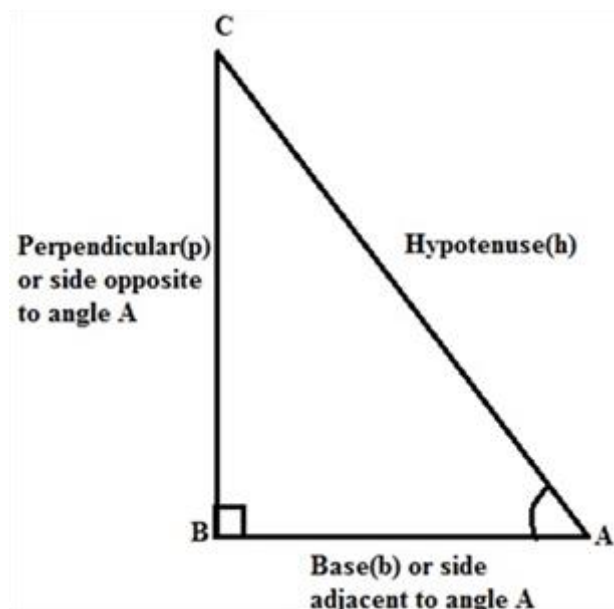
$$\tan (90^\circ - \theta) = \cot \theta$$

$$\cot (90^\circ - \theta) = \tan \theta$$

$$\operatorname{cosec} (90^\circ - \theta) = \sec \theta$$

$$\sec (90^\circ - \theta) = \operatorname{cosec} \theta$$

Finding Trigonometric Ratios of Complementary Angles



$\angle A$ and $\angle C$ form a complementary pair.

$$\Rightarrow \angle A + \angle C = 90^\circ$$

The relationship between the acute angle and the lengths of sides of a right-angle triangle is expressed by trigonometric ratios. For the given right angle triangle, the trigonometric ratios of $\angle A$ is given as follows:

$$\sin A = BC/AC$$

$$\cos A = AB/AC$$

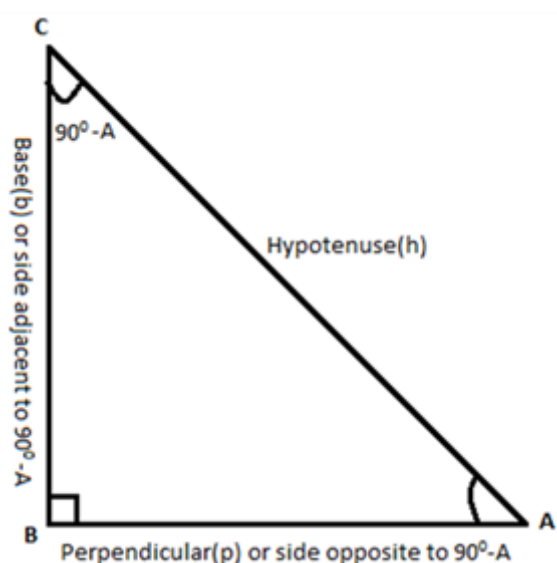
$$\tan A = BC/AB$$

$$\csc A = 1/\sin A = AC/BC$$

$$\sec A = 1/\cos A = AC/AB$$

$$\cot A = 1/\tan A = AB/BC$$

The trigonometric ratio of the complement of $\angle A$. It means that the $\angle C$ can be given as $90^\circ - \angle A$



As $\angle C = 90^\circ - A$ (A is used for convenience instead of $\angle A$), and the side opposite to $90^\circ - A$ is AB and the side adjacent to the angle $90^\circ - A$ is BC as shown in the figure given above.

Therefore,

$$\sin (90^\circ - A) = AB/AC$$

$$\cos (90^\circ - A) = BC/AC$$

$$\tan (90^\circ - A) = AB/BC$$

$$\csc (90^\circ - A) = 1/\sin (90^\circ - A) = AC/AB$$

$$\sec (90^\circ - A) = 1/\cos (90^\circ - A) = AC/BC$$

$$\cot (90^\circ - A) = 1/\tan (90^\circ - A) = BC/AB$$

Comparing the above set of ratios with the ratios mentioned earlier, it can be seen that;

$$\sin (90^\circ - A) = \cos A ; \cos (90^\circ - A) = \sin A$$

$$\tan (90^\circ - A) = \cot A ; \cot (90^\circ - A) = \tan A$$

$$\sec (90^\circ - A) = \csc A ; \csc (90^\circ - A) = \sec A$$

These relations are valid for all the values of A that lies between 0° and 90° .

19. Trigonometric Identities

Trigonometric Identities are useful whenever trigonometric functions are involved in an expression or an equation. Trigonometric Identities are true for every value of variables occurring on both sides of an equation. Geometrically, these identities involve certain trigonometric functions (such as sine, cosine, tangent) of one or more angles.

Sine, cosine and tangent are the primary trigonometry functions whereas cotangent, secant and cosecant are the other three functions. The trigonometric identities are based on all the six trig functions. Check Trigonometry Formulas to get formulas related to trigonometry.

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$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

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Express $\tan A$, $\cos A$ in terms of $\sin A$

Solution : Since, $\cos^2 A + \sin^2 A = 1$

$$\cos^2 A = 1 - \sin^2 A \text{ i.e. } \cos A = \sqrt{1 - \sin^2 A}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$$

Study of relationships between the sides & angles of a right triangle

$$\text{Sine of } \angle A = \frac{BC}{AC}$$

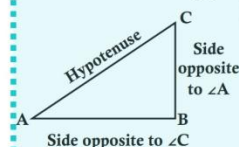
$$\text{Cosine of } \angle A = \frac{AB}{AC}$$

$$\text{Tangent of } \angle A = \frac{BC}{AB}$$

$$\text{Cosecant of } \angle A = \frac{AC}{BC}$$

$$\text{Secant of } \angle A = \frac{AC}{AB}$$

$$\text{Cotangent of } \angle A = \frac{AB}{BC}$$



Example

Trigonometry

Trigonometric Identities

Introduction to Trigonometry

Trigonometry Ratio

Complementary Angles

Values

$$\cos^2 A + \sin^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A; 0 \leq A \leq 90^\circ$$

$$\cot^2 A + 1 = \operatorname{cosec}^2 A; 0 \leq A \leq 90^\circ$$

$\sin(90^\circ - A) = \cos A$
 $\cos(90^\circ - A) = \sin A$
 $\tan(90^\circ - A) = \cot A$
 $\cot(90^\circ - A) = \tan A$
 $\sec(90^\circ - A) = \operatorname{cosec} A$
 $\operatorname{cosec}(90^\circ - A) = \sec A$

Note: How to learn the relation

"Some people have" $\sin \theta = \frac{P}{H}$

"Curly Brown Hair" $\cos \theta = \frac{B}{H}$

"through proper Brushing" $\tan \theta = \frac{P}{B}$

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not (∞) defined
$\operatorname{cosec} A$	Not (∞) defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not (∞) defined
$\cot A$	Not (∞) defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Important Questions

Multiple Choice questions-

1. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

- (a) $\cos \beta$
- (b) $\cos 2\beta$
- (c) $\sin \alpha$
- (d) $\sin 2\alpha$

2. If $\cos(40^\circ + A) = \sin 30^\circ$, the value of A is:?

- (a) 60°
- (b) 20°
- (c) 40°
- (d) 30°

3. If $\sin x + \operatorname{cosec} x = 2$, then $\sin^{19}x + \operatorname{cosec}^{20}x =$

- (a) 2^{19}
- (b) 2^{20}
- (c) 2
- (d) 2^{39}

4. If $\cos 9a = \sin a$ and $9a < 90^\circ$, then the value of $\tan 5a$ is

- (a) $\frac{1}{\sqrt{3}}$
- (b) $\sqrt{3}$
- (c) 1
- (d) 0

5. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ is equal to

- (a) 0
- (b) 1
- (c) 2

(d) -1

6. Ratios of sides of a right triangle with respect to its acute angles are known as

(a) trigonometric identities

(b) trigonometry

(c) trigonometric ratios of the angles

(d) none of these

7. The value of $\cos \theta \cos(90^\circ - \theta) - \sin \theta \sin(90^\circ - \theta)$ is:

(a) 1

(b) 0

(c) -1

(d) 2

8. If $x = a \cos \theta$ and $y = b \sin \theta$, then $b^2x^2 + a^2y^2 =$

(a) ab

(b) $b^2 + a^2$

(c) a^2b^2

(d) a^4b^4

9. If x and y are complementary angles, then

(a) $\sin x = \sin y$

(b) $\tan x = \tan y$

(c) $\cos x = \cos y$

(d) $\sec x = \operatorname{cosec} y$

10. $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$ is equal to

(a) $2 \cos \theta$

(b) 0

(c) $2 \sin \theta$

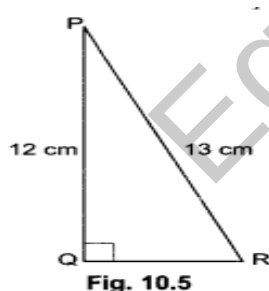
(d) 1

Very Short Questions:

1. Find maximum value of $\frac{1}{\sec \theta}$, $0^\circ \leq \theta \leq 90^\circ$.
2. Given that $\sin \theta = \frac{a}{b}$, find the value of $\tan \theta$.
3. If $\sin \theta = \cos \theta$, then find the value of $2 \tan \theta + \cos^2 \theta$.
4. If $\sin (x - 20)^\circ = \cos (3x - 10)^\circ$, then find the value of x .
5. If $\sin^2 A = \frac{1}{2} \tan^2 45^\circ$, where A is an acute angle, then find the value of A .
6. If $x = a \cos \theta$, $y = b \sin \theta$, then find the value of $b^2 x^2 + a^2 y^2 - a^2 b^2$.
7. If $\tan A = \cot B$, prove that $A + B = 90^\circ$.
8. If $\sec A = 2x$ and $\tan A = 2x$, find the value of $2(x^2 - \frac{1}{x^2})$.
9. In a $\triangle ABC$, if $\angle C = 90^\circ$, prove that $\sin^2 A + \sin^2 B = 1$.
10. If $\sec 4A = \operatorname{cosec} (A - 20^\circ)$ where $4A$ is an acute angle, find the value of A .

Short Questions :

1. If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.
2. Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.
3. In Fig. 10.5, find $\tan P - \cot R$.



4. If $\sin \theta + \cos \theta = \sqrt{3}$, then prove that $\tan \theta + \cot \theta = 1$.
5. Prove that $\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$

$$6. \frac{\sec^2 54^\circ - \cot^2 36^\circ}{\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ} + 2 \sin^2 38^\circ \cdot \sec^2 52^\circ - \sin^2 45^\circ.$$

$$7. \frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 50^\circ \cdot \tan 70^\circ}{5}$$

8.
$$\frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \left[\frac{\sin(90^\circ - \theta) \cdot \sin \theta}{\tan \theta} + \frac{\cos(90^\circ - \theta) \cdot \cos \theta}{\cot \theta} \right]$$
9. Evaluate: $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$.
10. Without using tables, evaluate the following:

$$3 \cos 68^\circ \cdot \operatorname{cosec} 22^\circ - \frac{1}{2} \tan 43^\circ \cdot \tan 47^\circ \cdot \tan 12^\circ \cdot \tan 60^\circ \cdot \tan 78^\circ$$

Long Questions :

1. In ΔPQR , right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

2. In triangle ABC right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$ find the value of:

(i) $\sin A \cos C + \cos A \sin C$ (ii) $\cos A \cos C - \sin A \sin C$.

3. If $\cot \theta = \frac{7}{8}$, evaluate:

(i)

$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

(ii) $\cot^2 \theta$

4. If $3 \cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

5. Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

6. Prove that

$$\left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A.$$

7. Prove that:

$$\tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}.$$

8. Prove that:

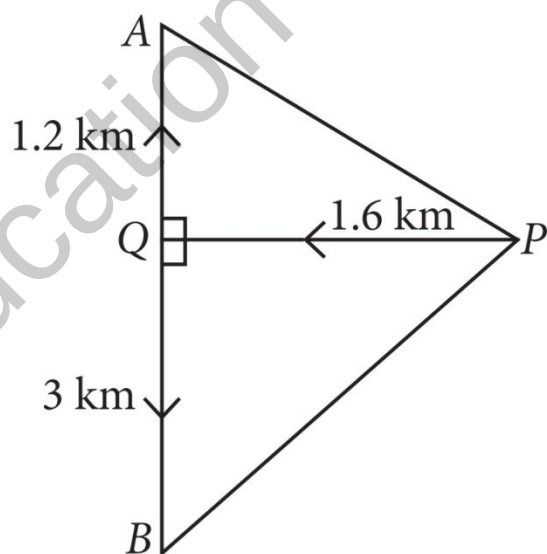
$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 + 2 \tan^2 A = 2 \sec^2 A.$$

9. Prove that: $(\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2 = (1 + \sec \theta \operatorname{cosec} \theta)^2$.
10. Prove that:

$$\frac{1}{(\operatorname{cosec} x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\operatorname{cosec} x - \cot x)}$$

Assertion Reason Questions-

1. Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400km/ hr and 500km/ hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes, then answer the following questions.



i. Find $\tan \theta$ if $\angle APQ = \theta$.

- a. $\frac{1}{2}$
- b. $\frac{1}{\sqrt{2}}$
- c. $\frac{\sqrt{3}}{2}$
- d. $\frac{3}{4}$

ii. Find $\cot B$.

- a. $\frac{3}{4}$
- b. $\frac{15}{4}$
- c. $\frac{3}{8}$
- d. $\frac{15}{8}$

iii. Find $\tan A$.

- a. 2
- b. $\sqrt{2}$
- c. $\frac{4}{3}$
- d. $\frac{2}{\sqrt{3}}$

iv. Find $\sec A$.

- a. 1
- b. $\frac{2}{3}$
- c. $\frac{4}{3}$
- d. $\frac{5}{3}$

v. Find cosec B.

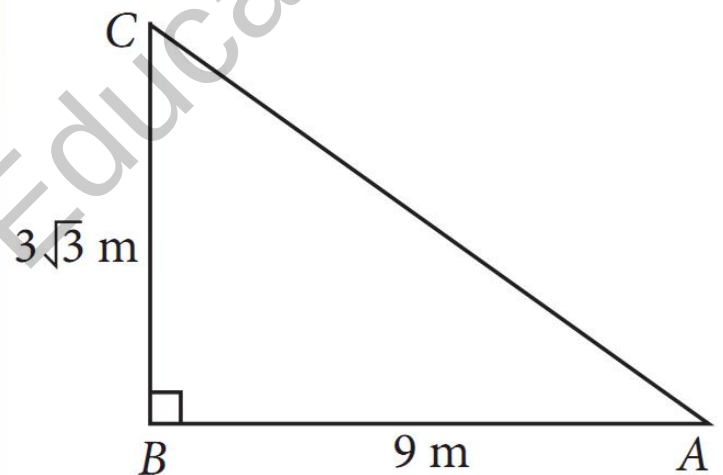
a. $\frac{17}{8}$

b. $\frac{12}{5}$

c. $\frac{5}{12}$

d. $\frac{8}{17}$

2. Three friends - Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right angled triangle such that $AB = 9$ m, $BC = 3\sqrt{3}$ m and $\angle B = 90^\circ$, then answer the following questions.



i. The measure of $\angle A$ is:

- a. 30°
- b. 45°
- c. 60°
- d. None of these.

ii. The measure of $\angle C$ is:

- a. 30°
- b. 45°
- c. 60°
- d. None of these.

iii. The length of AC is:

- a. $2\sqrt{3}$ m
- b. $\sqrt{3}$ m
- c. $4\sqrt{3}$ m
- d. $6\sqrt{3}$ m

iv. $\cos 2A =$

- a. 0
- b. $\frac{1}{2}$
- c. $\frac{1}{\sqrt{2}}$
- d. $\frac{\sqrt{3}}{2}$

v. $\sin\left(\frac{C}{2}\right) =$

- a. 0
- b. $\frac{1}{2}$
- c. $\frac{1}{\sqrt{2}}$
- d. $\frac{\sqrt{3}}{2}$

Assertion Reason Questions-

1. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

- a. A is true, R is true; R is a correct explanation for A.
- b. A is true, R is true; R is not a correct explanation for A.
- c. A is true; R is false.
- d. A is false; R is true.

Assertion: The value of each of the trigonometric ratios of an angle does not depend on the size of the triangle. It only depends on the angle.

Reason: In right $\triangle ABC$, $\angle B = 90^\circ$ and $\angle A = \theta^\circ$ $\sin \theta = \frac{BC}{AC} < 1$ and $\cos \theta = \frac{AB}{AC} < 1$ as hypotenuse is the longest side.

2. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

- a. A is true, R is true; R is a correct explanation for A.
- b. A is true, R is true; R is not a correct explanation for A.
- c. A is true; R is false.
- d. A is false; R is true.

Assertion: $\sin 60^\circ = \cos 30^\circ$

Reason: $\sin 2\theta = \sin \theta$ where θ is an acute angle.

Answer Key-

Multiple Choice questions-

- 1. (b) $\cos 2\beta$
- 2. (b) 20°
- 3. (c) 2
- 4. (c) 1

5. (c) 2
6. (c) trigonometric ratios of the angles
7. (b) 0
8. (c) a^2b^2
9. (d) $\sec x = \operatorname{cosec} y$
10. (b) 0

Very Short Answer :

1. $\frac{1}{\sec \theta}$, $(0^\circ \leq \theta \leq 90^\circ)$ (Given)

$\therefore \sec \theta$ is in the denominator

$\therefore$ The min. value of $\sec \theta$ will return max. value for $\frac{1}{\sec \theta}$.

But the min. value of $\sec \theta$ is $\sec 0^\circ = 1$.

Hence, the max. value of $\frac{1}{\sec \theta} = \frac{1}{1} = 1$

2. $\sin \theta = \frac{a}{b}$

$$\Rightarrow \cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{\frac{b^2 - a^2}{b^2}} = \frac{\sqrt{b^2 - a^2}}{b}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{a}{b}}{\frac{\sqrt{b^2 - a^2}}{b}} = \frac{a}{\sqrt{b^2 - a^2}}$$

3. $\sin \theta = \cos \theta$ (Given)

It means value of $\theta = 45^\circ$

Now, $2 \tan \theta + \cos^2 \theta = 2 \tan 45^\circ + \cos^2 45^\circ$

4. $\sin (x - 20)^\circ = \cos (3x - 10)^\circ$

$$\Rightarrow \cos [90^\circ - (x - 20)^\circ] = \cos (3x - 10)^\circ$$

By comparing the coefficient

$$90^\circ - x^\circ + 20^\circ = 3x^\circ - 10^\circ = 110^\circ + 10^\circ = 3x^\circ + x^\circ$$

$$120^\circ = 4x^\circ$$

$$\Rightarrow \frac{120^\circ}{4} = 30^\circ$$

5. $\sin^2 A = 12 \tan^2 45^\circ$

$$\Rightarrow \sin^2 A = \frac{1}{2} (1)^2 [\because \tan 45^\circ = 1]$$

$$= \sin^2 A = \frac{1}{2}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{2}}$$

Hence, $\angle A = 45^\circ$

6. Given $x = a \cos \theta$, $y = b \sin \theta$

$$b^2 x^2 + a^2 y^2 - a^2 b^2 = b^2 (a \cos \theta)^2 + a^2 (b \sin \theta)^2 - a^2 b^2$$

$$= a^2 b^2 \cos^2 \theta + a^2 b^2 \sin^2 \theta - a^2 b^2 = a^2 b^2 (\sin^2 \theta + \cos^2 \theta) - a^2 b^2$$

$$= a^2 b^2 - a^2 b^2 = 0 (\because \sin^2 \theta + \cos^2 \theta = 1)$$

7. We have

$$\tan A = \cot B$$

$$\Rightarrow \tan A = \tan (90^\circ - B)$$

$$A = 90^\circ - B$$

$$[\because \text{Both } A \text{ and } B \text{ are acute angles}]$$

$$\Rightarrow A + B = 90^\circ$$

8.

$$2 \left(x^2 - \frac{1}{x^2} \right) = 2 \left(\frac{\sec^2 A}{4} - \frac{\tan^2 A}{4} \right) = \frac{2}{4} (\sec^2 A - \tan^2 A) = \frac{1}{2} \times 1 = \frac{1}{2}$$

9. Since $\angle C = 90^\circ$

$$\therefore \angle A + \angle B = 180^\circ - \angle C = 90^\circ$$

$$\text{Now, } \sin^2 A + \sin^2 B = \sin^2 A + \sin^2 (90^\circ - A) = \sin^2 A + \cos^2 A = 1$$

10. We have

$$\sec 4A = \operatorname{cosec} (A - 20^\circ)$$

$$\Rightarrow \operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 20^\circ)$$

$$\therefore 90^\circ - 4A = A - 20^\circ$$

$$\Rightarrow 90^\circ + 20^\circ = A + 4A$$

$$\Rightarrow 110^\circ = 5A$$

$$\therefore A = \frac{110}{5} = 22^\circ$$

Short Answer :

- Let us first draw a right $\triangle ABC$ in which $\angle C = 90^\circ$.

Now, we know that

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{3}{4}$$

Let $BC = 3k$ and $AB = 4k$, where k is a positive number.

Then, by Pythagoras Theorem, we have

$$\begin{aligned} AB^2 &= BC^2 + AC^2 & \Rightarrow & (4k)^2 = (3k)^2 + AC^2 \\ \Rightarrow 16k^2 - 9k^2 &= AC^2 & \Rightarrow & 7k^2 = AC^2 \end{aligned}$$

$$\therefore AC = \sqrt{7}k$$

$$\therefore \cos A = \frac{AC}{AB} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4} \quad \text{and} \quad \tan A = \frac{BC}{AC} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

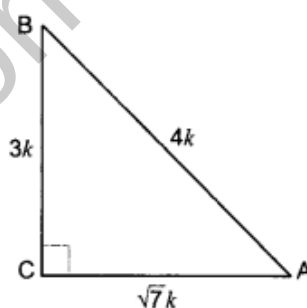


Fig. 10.3

- Let us first draw a right $\triangle ABC$ in which $\angle B = 90^\circ$.

Now, we have, $15 \cot A = 8$

$$\therefore \cot A = \frac{8}{15} = \frac{AB}{BC} = \frac{\text{Base}}{\text{Perpendicular}}$$

Let $AB = 8k$ and $BC = 15k$

Then, $AC = \sqrt{(AB)^2 + (BC)^2}$ (By Pythagoras Theorem)

$$= \sqrt{(8k)^2 + (15k)^2} = \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$$

$$\therefore \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

$$\text{and, } \sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$

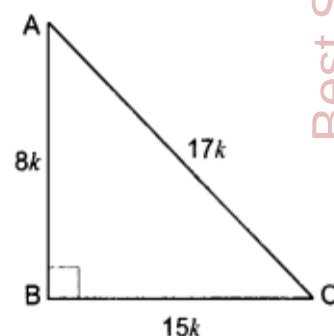


Fig. 10.4

- Using Pythagoras Theorem, we have

$$PR^2 = PO^2 + QR^2$$

$$\Rightarrow (13)^2 = (12)^2 + QR^2$$

$$\Rightarrow 169 = 144 + QR^2$$

$$\Rightarrow QR^2 = 169 - 144 = 25$$

$$\Rightarrow QR = 5 \text{ cm}$$

$$\text{Now, } \tan P = \frac{QR}{PQ} = \frac{5}{12} \text{ and } \cot R = \frac{QR}{PQ} = \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

4. $\sin \theta + \cos \theta = \sqrt{3}$

$$\Rightarrow (\sin \theta + \cos \theta)^2 = 3$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 3$$

$$\Rightarrow 2 \sin \theta \cos \theta = 2 \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$\Rightarrow \sin \theta \cdot \cos \theta = 1 = \sin^2 \theta + \cos^2 \theta$$

$$\Rightarrow 1 = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\Rightarrow 1 = \tan \theta + \cot \theta = 1$$

$$\text{Therefore } \tan \theta + \cot \theta = 1$$

5.

$$\begin{aligned} \text{LHS} &= \frac{1 - \sin \theta}{1 + \sin \theta} \\ &= \frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} \quad [\text{Rationalising the denominator}] \\ &= \frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta} = \left(\frac{1 - \sin \theta}{\cos \theta} \right)^2 = \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right)^2 \\ &= (\sec \theta - \tan \theta)^2 = \text{RHS} \end{aligned}$$

6.

$$\begin{aligned}
 \text{We have, } & \frac{\sec^2 54^\circ - \cot^2 36^\circ}{\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ} + 2 \sin^2 38^\circ \cdot \sec^2 52^\circ - \sin^2 45^\circ \\
 &= \frac{\sec^2 (90^\circ - 36^\circ) - \cot^2 36^\circ}{\operatorname{cosec}^2 (90^\circ - 33^\circ) - \tan^2 33^\circ} + 2 \sin^2 38^\circ \cdot \sec^2 (90^\circ - 38^\circ) - \sin^2 45^\circ \\
 &= \frac{\operatorname{cosec}^2 36^\circ - \cot^2 36^\circ}{\sec^2 33^\circ - \tan^2 33^\circ} + 2 \sin^2 38^\circ \cdot \operatorname{cosec}^2 38^\circ - \left(\frac{1}{\sqrt{2}}\right)^2 \\
 &= \frac{1}{1} + 2 \cdot 1 - \frac{1}{2} = 3 - \frac{1}{2} = \frac{5}{2}
 \end{aligned}$$

7.

$$\begin{aligned}
 \text{We have, } & \frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 50^\circ \cdot \tan 70^\circ}{5} \\
 &= \frac{2 \sin (90^\circ - 22^\circ)}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan (90^\circ - 15^\circ)} \\
 &\quad - \frac{3 \tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \tan (90^\circ - 40^\circ) \cdot \tan (90^\circ - 20^\circ)}{5} \\
 &= \frac{2 \cos 22^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \cot 15^\circ} - \frac{3 \tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \cot 40^\circ \cdot \cot 20^\circ}{5} \\
 &= 2 - \frac{2}{5} - \frac{3 \tan 45^\circ \cdot (\tan 20^\circ \cdot \cot 20^\circ) \cdot (\tan 40^\circ \cdot \cot 40^\circ)}{5} \\
 &= 2 - \frac{2}{5} - \frac{3}{5} \cdot 1 \cdot 1 \cdot 1 = 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1
 \end{aligned}$$

8.

$$\begin{aligned}
 \text{We have } & \frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \left[\frac{\sin (90^\circ - \theta) \cdot \sin \theta}{\tan \theta} + \frac{\cos (90^\circ - \theta) \cdot \cos \theta}{\cot \theta} \right] \\
 &= \frac{\sin^2 20^\circ + \sin^2 (90^\circ - 20^\circ)}{\cos^2 20^\circ + \cos^2 (90^\circ - 20^\circ)} + \left[\frac{\cos \theta \cdot \sin \theta}{\tan \theta} + \frac{\cos \theta \cdot \sin \theta}{\cot \theta} \right] \\
 &= \frac{\sin^2 20^\circ + \cos^2 20^\circ}{\cos^2 20^\circ + \sin^2 20^\circ} + \left[\frac{\cos \theta \cdot \sin \theta}{\frac{\sin \theta}{\cos \theta}} + \frac{\cos \theta \cdot \sin \theta}{\frac{\cos \theta}{\sin \theta}} \right] \\
 &= \frac{1}{1} + [\cos^2 \theta + \sin^2 \theta] = 1 + 1 = 2.
 \end{aligned}$$

$$9. \quad \sin 25^\circ \cdot \cos 65^\circ + \cos 25^\circ \cdot \sin 65^\circ$$

$$= \sin (90^\circ - 65^\circ) \cdot \cos 65^\circ + \cos (90^\circ - 65^\circ) \cdot \sin 65^\circ$$

$$= \cos 65^\circ \cdot \cos 65^\circ + \sin 65^\circ \cdot \sin 65^\circ$$

$$= \cos^2 65^\circ + \sin^2 65^\circ = 1.$$

10. We have,

$$3 \cos 68^\circ \cdot \operatorname{cosec} 22^\circ - \frac{1}{2} \tan 43^\circ \cdot \tan 47^\circ \cdot \tan 12^\circ \cdot \tan 60^\circ \cdot \tan 78^\circ.$$

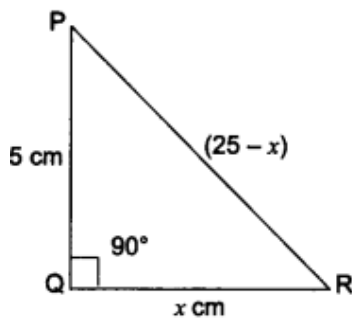
$$= 3 \cos (90^\circ - 22^\circ) \cdot \operatorname{cosec} 22^\circ - \frac{1}{2} \cdot \{\tan 43^\circ \cdot \tan (90^\circ - 43^\circ)\} \cdot \{\tan 12^\circ \cdot \tan (90^\circ - 12^\circ) \cdot \tan 60^\circ\}$$

$$= 3 \sin 22^\circ \cdot \operatorname{cosec} 22^\circ - \frac{1}{2} (\tan 43^\circ \cdot \cot 43^\circ) \cdot (\tan 12^\circ \cdot \cot 12^\circ) \cdot \tan 60^\circ$$

$$= 3 \times 1 - \frac{1}{2} \times 1 \times 1 \times \sqrt{3} = 3 - \frac{\sqrt{3}}{2} = \frac{6 - \sqrt{3}}{2}.$$

Long Answer :

1.



We have a right-angled ΔPQR in which $\angle Q = 90^\circ$.

Let $QR = x$ cm

Therefore, $PR = (25 - x)$ cm

By Pythagoras Theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$(25 - x)^2 = 5^2 + x^2$$

$$= (25 - x)^2 - x^2 = 25$$

$$(25 - x - x)(25 - x + x) = 25$$

$$(25 - 2x) 25 = 25$$

$$25 - 2x = 1$$

$$25 - 1 = 2x$$

$$= 24 = 2x$$

$$\therefore x = 12 \text{ cm}$$

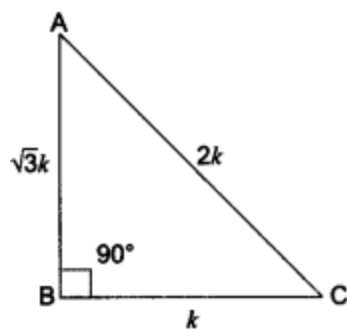
Hence, $QR = 12$ cm

$PR = (25 - x)$ cm $= 25 - 12 = 13$ cm

$PQ = 5$ cm

$$\therefore \sin P = \frac{QR}{PR} = \frac{12}{13}; \quad \cos P = \frac{PQ}{PR} = \frac{5}{13}; \quad \tan P = \frac{QR}{PQ} = \frac{12}{5} \text{ cm}$$

2.



We have a right-angled $\triangle ABC$ in which $\angle B = 90^\circ$.

and, $\tan A = \frac{1}{\sqrt{3}}$

Now, $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

Let $BC = k$ and $AB = \sqrt{3}k$

$\therefore$ By Pythagoras Theorem, we have

$$\Rightarrow AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2$$

$$\Rightarrow AC^2 = 4k^2$$

$$\text{Now, } \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{k}{2k} = \frac{1}{2}; \quad \cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}; \quad \cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{k}{2k} = \frac{1}{2}$$

$$(i) \sin A \cdot \cos C + \cos A \cdot \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1.$$

$$(ii) \cos A \cdot \cos C - \sin A \cdot \sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0.$$

3. Let us draw a right triangle ABC in which $\angle B = 90^\circ$ and $\angle C = \theta$.

We have, $\cot \theta = \frac{7}{8} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB}$ (given)

Let $BC = 7k$ and $AB = 8k$

Therefore, by Pythagoras Theorem

$$AC^2 = AB^2 + BC^2 = (8k)^2 + (7k)^2 = 64k^2 + 49k^2$$

$$AC^2 = 113k^2 \quad \therefore AC = \sqrt{113}k$$

$$\therefore \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\text{and } \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

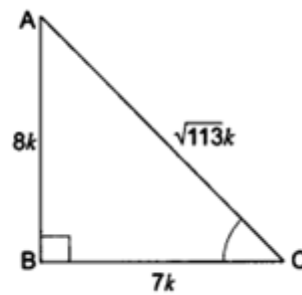


Fig. 10.9

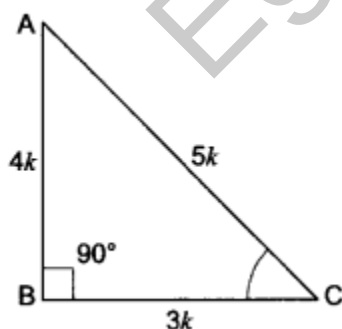
$$\begin{aligned} (i) \quad \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} &= \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} = \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} \\ &= \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}} = \frac{49}{64} \end{aligned}$$

Alternate method:

$$(i) \quad \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} = \cot^2 \theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

$$(ii) \quad \cot^2 \theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

4.



Let us consider a right triangle ABC in which $\angle B = 90^\circ$

$$\text{Now, } \cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{4}{3}$$

Let $AB = 4k$ and $BC = 3k$

∴ By Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$AC = (4k)^2 + (3k)^2 = 16k^2 + 9k^2$$

$$AC^2 = 25k^2$$

$$\therefore AC = 5k$$

$$\text{Therefore, } \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

$$\text{and, } \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

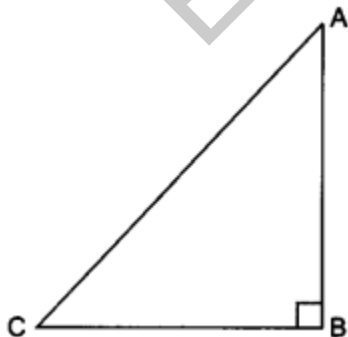
$$\text{Now, LHS} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{16 - 9}{16 + 9} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\text{Hence, } \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A.$$

5.



Let us consider a right-angled $\triangle ABC$ in which $\angle B = 90^\circ$.

For $\angle A$ we have

$$\therefore \sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB}$$

$$\Rightarrow \frac{\sec A}{1} = \frac{AC}{AB} \quad \Rightarrow \quad AC = AB \sec A$$

Let $AB = k$ and $AC = k \sec A$

$\therefore$ By Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2 \quad \Rightarrow \quad k^2 \sec^2 A = k^2 + BC^2$$

$$\therefore BC^2 = k^2 \sec^2 A - k^2 \quad \Rightarrow \quad BC = k \sqrt{\sec^2 A - 1}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k \sqrt{\sec^2 A - 1}}{k \sec A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cos A = \frac{AB}{AC} = \frac{k}{k \sec A} = \frac{1}{\sec A}$$

$$\tan A = \frac{BC}{AB} = \frac{k \sqrt{\sec^2 A - 1}}{k} = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{AC}{BC} = \frac{k \sec A}{k \sqrt{\sec^2 A - 1}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

6.

$$\begin{aligned} \text{LHS} &= \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \frac{\sec^2 A}{\operatorname{cosec}^2 A} \\ &= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}} \right)^2 \\ &= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}} \right)^2 = \left(\frac{1 - \tan A}{\tan A - 1} \times \tan A \right)^2 = (-\tan A)^2 = \tan^2 A \end{aligned}$$

$$\text{LHS} = \text{RHS.}$$

7.

$$\begin{aligned}
 \text{LHS} &= \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B} \\
 &= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cos^2 B} = \frac{(1 - \cos^2 A) \cos^2 B - \cos^2 A (1 - \cos^2 B)}{\cos^2 A \cos^2 B} \\
 &= \frac{\cos^2 B - \cos^2 A \cos^2 B - \cos^2 A + \cos^2 A \cos^2 B}{\cos^2 A \cos^2 B} = \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}
 \end{aligned}$$

$$\begin{aligned}
 \text{Also } \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} &= \frac{(1 - \sin^2 B) - (1 - \sin^2 A)}{\cos^2 A \cos^2 B} \\
 &= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \text{RHS.}
 \end{aligned}$$

8.

$$\begin{aligned}
 \text{LHS} &= \frac{\operatorname{cosec} A}{(\operatorname{cosec} A - 1)} + \frac{\operatorname{cosec} A}{(\operatorname{cosec} A + 1)} \\
 &= \frac{\operatorname{cosec} A (\operatorname{cosec} A + 1) + \operatorname{cosec} A (\operatorname{cosec} A - 1)}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)} \\
 &= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{(\operatorname{cosec}^2 A - 1)} = \frac{2 \operatorname{cosec}^2 A}{1 + \cot^2 A - 1} = \frac{2 \operatorname{cosec}^2 A}{\cot^2 A}
 \end{aligned}$$

$$= 2 \operatorname{cosec}^2 A \tan^2 A = 2(1 + \cot^2 A) \cdot \tan^2 A$$

$$= 2 \tan^2 A + 2 \tan^2 A \cdot \cot^2 A (\because \tan A \cot A = 1)$$

$$= 2 + 2 \tan^2 A = 2(1 + \tan^2 A) = 2 \sec^2 A = \text{RHS.}$$

9.

$$\text{LHS} = (\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2$$

$$= \left(\sin \theta + \frac{1}{\cos \theta} \right)^2 + \left(\cos \theta + \frac{1}{\sin \theta} \right)^2 = \left(\frac{\sin \theta \cos \theta + 1}{\cos \theta} \right)^2 + \left(\frac{\cos \theta \sin \theta + 1}{\sin \theta} \right)^2$$

$$= \frac{(\sin \theta \cos \theta + 1)^2}{\cos^2 \theta} + \frac{(\cos \theta \sin \theta + 1)^2}{\sin^2 \theta} = (\sin \theta \cos \theta + 1)^2 \left(\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \right)$$

$$= (\sin \theta \cos \theta + 1)^2 \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta} \right) = (\sin \theta \cos \theta + 1)^2 \cdot \left(\frac{1}{\cos^2 \theta \sin^2 \theta} \right)$$

$$= \left(\frac{\sin \theta \cos \theta + 1}{\cos \theta \sin \theta} \right)^2 = \left(1 + \frac{1}{\cos \theta \sin \theta} \right)^2$$

$$= (1 + \sec \theta \operatorname{cosec} \theta)^2 = \text{RHS.}$$

$$= (1 + \sec \theta \operatorname{cosec} \theta)^2 = \text{RHS.}$$

10. In order to show that,

It is sufficient to show

$$\frac{1}{\operatorname{cosec} x + \cot x} + \frac{1}{\operatorname{cosec} x - \cot x} = \frac{1}{\sin x} + \frac{1}{\sin x}$$

$$\Rightarrow \frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{2}{\sin x} \quad \dots(i)$$

Now, LHS of above is

$$\begin{aligned} & \frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} \\ &= \frac{(\operatorname{cosec} x - \cot x) + (\operatorname{cosec} x + \cot x)}{(\operatorname{cosec} x - \cot x)(\operatorname{cosec} x + \cot x)} \\ &= \frac{2 \operatorname{cosec} x}{\operatorname{cosec}^2 x - \cot^2 x} \\ &= \frac{2 \operatorname{cosec} x}{1} = \frac{2}{\sin x} = \text{RHS of (i)} \end{aligned}$$

$$[\because (a+b)(a-b) = a^2 - b^2]$$

$$\text{Hence, } \frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{1}{\sin x} + \frac{1}{\sin x}$$

$$\text{or } \frac{1}{(\operatorname{cosec} x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\operatorname{cosec} x - \cot x)}.$$

Case Study Answers:

1. Answer :

i. (d) $\frac{3}{4}$

Solution:

$$\text{In } \triangle APQ, \tan \theta = \frac{AQ}{PQ} = \frac{1.2}{1.6} = \frac{3}{4}$$

ii. (d) $\frac{15}{8}$

Solution:

$$\text{In } \triangle PBQ, \cot B = \frac{QB}{PQ} = \frac{3}{1.6} = \frac{15}{8}$$

iii. (c) $\frac{4}{3}$

Solution:

$$\text{In } \triangle APQ, \tan A = \frac{PQ}{AQ} = \frac{1.6}{1.2} = \frac{4}{3}$$

iv. (d) $\frac{5}{3}$

Solution:

$$\text{We have, } \tan^2 A + 1 = \sec^2 A$$

$$\Rightarrow \sqrt{\left(\frac{4}{3}\right)^2 + 1}$$

$$= \sqrt{\frac{16}{9} + 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

v. (a) $\frac{17}{8}$

Solution:

$$\text{Since, } \operatorname{cosec} B = \sqrt{\cot^2 B + 1}$$

$$= \sqrt{\left(\frac{15}{8}\right)^2 + 1}$$

$$= \frac{17}{8}$$

2. Answer :

i. (a) 30°

Solution:

We have, $AB = 9$ m, $BC = \sqrt{3}$ m in $\triangle ABC$, we have

$$\tan A = \frac{BC}{AB} = \frac{\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan A = \tan 30^\circ \Rightarrow \angle A = 30^\circ$$

ii. (c) 60°

Solution:

$$\text{Similarly, } \tan C = \frac{AB}{BC} = \frac{9}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \tan C = \tan 60^\circ \Rightarrow \angle C = 60^\circ$$

iii. (d) $6\sqrt{3}$ m

Solution:

$$\text{Since, } \sin A = \frac{BC}{AC} \Rightarrow \sin 30^\circ = \frac{BC}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{\sqrt{3}}{AC} \Rightarrow AC = 2\sqrt{3} \text{ m}$$

iv. (b) $\frac{1}{2}$

Solution:

$$\therefore \angle A = 30^\circ$$

$$\therefore \cos 2A = \cos(2 \times 30^\circ) = \cos 60^\circ = \frac{1}{2}$$

v. (b) $\frac{1}{2}$

Solution:

$$\therefore \angle C = 60^\circ$$

$$\therefore \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60^\circ}{2}\right) = \sin 30^\circ = \frac{1}{2}$$

Assertion Reason Answer-

1. (b) A is true, R is true; R is not a correct explanation for A.
2. (c) A is true; R is false.