

# MATHEMATICS

## Chapter 6: LINEAR INEQUALITIES



## LINEAR INEQUALITIES

### Some Important Results

1. If 'k' is a positive real number, then  $|x| < k \Leftrightarrow -k < x < k, x \in (-k, k)$ .
2. If 'k' is a positive real number, then  $|x| \leq k \Leftrightarrow -k \leq x \leq k, x \in [-k, k]$ .
3. If 'k' is a positive real number, then  $|x| > k \Leftrightarrow x < -k, x > k$ .
4. If 'k' is a positive real number, then  $|x| \geq k \Leftrightarrow x \leq -k, x \geq k$ .
5. If 'k' is a positive real number and 'y' is a fixed number, then  $|x - y| < k \Leftrightarrow y - k < x < y + k, x \in (y - k, y + k)$ .
6. If 'k' is a positive real number and 'y' is a fixed number, then  $|x - y| \leq k \Leftrightarrow y - k \leq x \leq y + k, x \in [y - k, y + k]$ .
7. If 'k' is a positive real number and 'y' is a fixed number, then  $|x - y| > k \Leftrightarrow x < y - k \text{ or } x > y + k$ .
8. If 'k' be a positive real number and 'y' is a fixed number, then  $|x - y| \geq k \Leftrightarrow x \leq y - k \text{ or } x \geq y + k$ .
9. If a, b and c are positive real numbers, then  $a < |x| < b \Leftrightarrow x \in (-b, a) \cup (a, b)$ .
10. If a, b and c are positive real numbers, then  $a \leq |x| \leq b \Leftrightarrow x \in [-b, a] \cup (a, b)$ .
11. If a, b and c are positive real numbers, then  $a \leq |x - c| \leq b \Leftrightarrow x \in [-b + c, -a + c] \cup [a + c, b + c]$ .
12. If a, b and c are positive real numbers, then  $a < |x - c| < b \Leftrightarrow x \in (-b + c, -a + c) \cup (a + c, b + c)$ .

### Definitions

1. Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an inequality.
2. Inequalities containing '<', or '>' are called strict inequalities.
3. Inequalities containing '≤' or '≥' are called slack inequalities.
4. An inequality containing any two of '<', '>', '≤' or '≥' is called double inequality.
5. Solution an inequality in one variable is the value of the variable which makes it a true statement.
6. Solving an inequality is the process of getting all possible solutions of an inequality.
7. Solution set is the set of all possible solutions of an inequality is known as its solution set.
8. A linear expression in one variable involving the inequality symbol is linear inequality in one variable.

General forms

$$ax + b < 0$$

$$ax + b > 0$$

$$ax + b \leq 0$$

$$ax + b \geq 0$$

9. A linear inequality involving two variables is known as a linear inequality in two variables.

General forms

$$ax + by < c$$

$$ax + by > c$$

$$ax + by \leq c$$

$$ax + by \geq c$$

10. The following is the example of quadratic inequalities known as quadratic inequalities

$$ax^2 + bx + c \leq 0$$

$$ax^2 + bx + c \geq 0$$

11. The inequalities of the form ' $<$ ' or ' $>$ ' are known as strict inequalities, whereas if they are of the form ' $\leq$ ' or ' $\geq$ ' then they are called slack inequalities.
12. The region containing all the solutions of an inequality is called the solution region.
13. The solution region of the system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.
14. Quadratic inequality is the quadratic polynomial with an inequality sign. Generic quadratic inequality is of the form  $ax^2 + bx + c > 0$ .

### Concepts

- If two real numbers are related by the symbols ' $<$ ', ' $>$ ', ' $\leq$ ' or ' $\geq$ ', then the inequality is a numerical inequality and in case of algebraic expressions it is literal inequality.  
 $2 < 3$  is numerical inequality.  
 $5x + 2 \leq 7$  is literal inequality.

### 2. Rules for simplifying the inequalities

**Rule 1:** Equal numbers may be added to (or subtracted from) both sides of an equation.

If  $a < b$ , then  $a + c < b + c$ .

**Rule 2:** Both sides of an equation may be multiplied (or divided) by the same non-zero number.

If  $a < b$ , then  $ac < bc$

**Rule 3:** Sign of inequality is reversed in case of multiplication (or division) by a negative number.

If  $a < b$ , then  $ak > bk$ , where  $k$  is a negative number.

**Rule 4:** Sign of inequality is reversed in case of taking the reciprocals.

3. A linear inequality can be solved by the following steps:

1. Obtain the linear inequality.

2. Group all variable terms on one side of the inequality and transpose the constant term on the other side of the inequality.

3. Simplify both the sides of the inequality to the standard form

$ax < b$ , or  $ax \leq b$ , or  $ax > b$ , or  $ax \geq b$

4. Solve the inequality by dividing both the sides of the inequality by the coefficient of the variable.

5. Write and depict the solution set in the form of number line.

4. **Some examples of graphs of linear inequality:** A linear inequality in one variable can be represented graphically as follows:

Representation of  $x \leq 1$



Representation of  $x \geq 1$



Representation of  $x > 1$



5. Steps to solve inequalities of the type,  $\frac{ax+b}{cx+d} > k$ ,  $\frac{ax+b}{cx+d} \geq k$ ,  $\frac{ax+b}{cx+d} < k$  and  $\frac{ax+b}{cx+d} \leq k$

Step 1: Transpose all the variable terms on LHS.

Step 2: Transpose all the variable terms on LHS.

Step 3: Simplify the terms in LHS.

Step 4: Make the coefficient of  $x$  positive in the numerator and denominator.

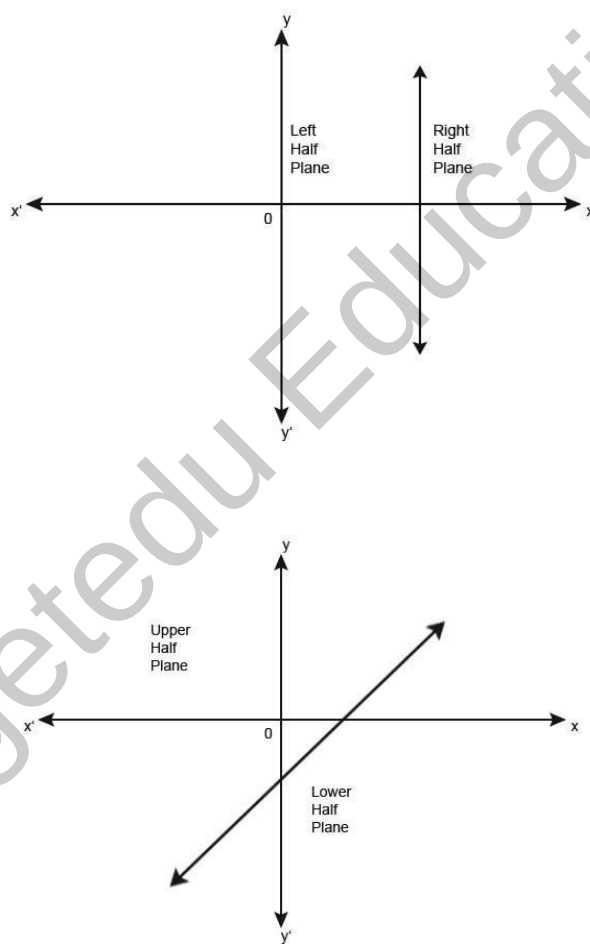
Step 5: Equate both the numerator and denominator to zero and find the values of  $x$  and critical points.

Step 6: Plot these critical points on the number line, which will divide the real line into three regions.

Step 7: Mark the sign of the functions over the respective intervals.

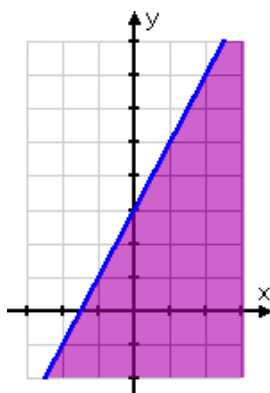
Step 8: Write solution regions in the form of intervals to get the required solution sets of the given inequality.

6. A linear inequality in two variables represents a half plane geometrically. Types of half planes.



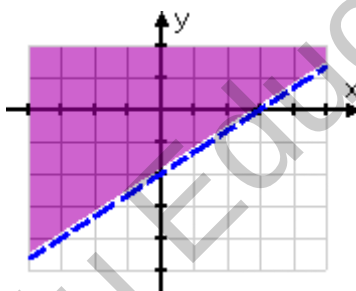
7. In order to identify the half plane represented by an inequality, take any point  $(a, b)$  (not on line) and check whether it satisfies the inequality or not. If it satisfies the inequality, then the inequality represents the half plane and subsequently shades the region which contains the point; otherwise, the inequality represents the half plane which does not contain the point within it. For convenience, the point  $(0, 0)$  is preferred.

8. If an inequality is of the type  $ax + b \geq c$  or  $ax + b \leq c$ , i.e. slack inequality, then the points on the line  $ax + b = c$  are also included in the solution region.



Solution of slack inequality

9. If an inequality is of the form  $ax + by > c$  or  $ax + by < c$ , then the points on the line  $ax + by = c$  are not to be included in the solution region.



Solution of strict inequality

10. To represent  $x < a$  (or  $x > a$ ) on a number line, put a circle on the number  $a$  and dark line to the left (or right) of the number  $a$ .
11. To represent  $x \leq a$  (or  $x \geq a$ ) on a number line, put a dark circle on the number  $a$  and dark line to the left (or right) of the number  $x$ .
12. Steps to represent the linear inequality in two variables graphically
- Step 1: Rewrite the inequality as linear equation, i.e.  $ax + by = c$ .
  - Step 2: Put  $x = 0$  to get y-intercept of the line, i.e.  $(0, c/b)$ .
  - Step 3: Put  $y = 0$  to get x-intercept of the line, i.e.  $(c/a, 0)$ .
  - Step 4: Join the two points, each on X-axis and Y-axis to get the graphical representation of the line.
  - Step 5: Choose a point  $(x_1, y_1)$  in one of the planes, i.e. either to the left or right or upper or lower half of the line, but not on the line.

Step 6: If  $(x_1, y_1)$  satisfies the given inequality, then the required region is that particular half plane in which  $(x_1, y_1)$  lie.

On the other hand, if  $(x_1, y_1)$  does not satisfy the given inequality, then the required solution region is the half plane which does not contain  $(x_1, y_1)$ .

13. Linear inequalities represent regions; regions common to the given inequalities will be the solution region.

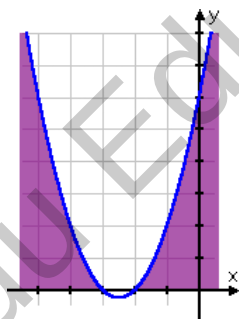
Similar to linear equations, there can be cases of overlapping of regions or no common regions for the given inequalities.

14. To solve a system of inequalities graphically

- Change the sign of equality to inequality and draw the graph of each line.
- Shade the region for each inequality.
- Common region to all the inequalities is the solution.

15. A linear inequality divides the plane into two half planes, while a quadratic inequality is represented by a parabola which divides the plane into different regions.

Region represented by the inequality  $x^2 + 5x + 6 \geq 0$



## 16. Interval Notations

Open Interval: The interval which contains all the elements between  $a$  and  $b$  excluding  $a$  and  $b$ . In set notations:

$$(a, b) = \{x : a < x < b\}$$



Closed interval: The interval which contains all the elements between  $a$  and  $b$  and also the end points  $a$  and  $b$  is called the **closed interval**.

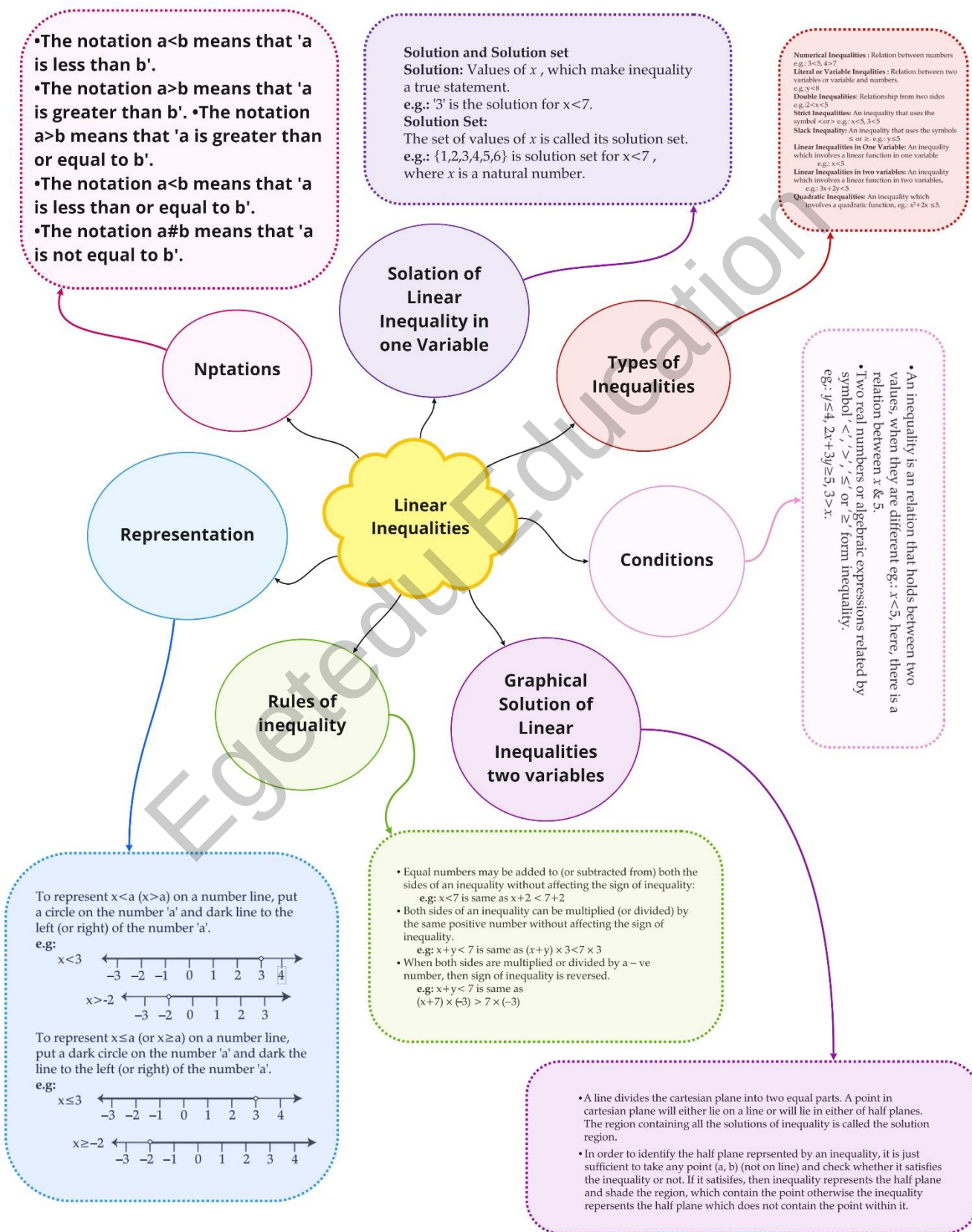
$$[a, b] = \{x : a \leq x \leq b\}$$



Semi open intervals:

$(a, b] = \{x : a < x \leq b\}$  includes all the elements from  $a$  to  $b$  including  $a$  and excluding  $b$ .

$[a, b) = \{x : a \leq x < b\}$  includes all the elements from  $a$  to  $b$  excluding  $a$  and including  $b$ .



## Important Questions

### Multiple Choice questions-

Question 1. If  $-2 < 2x - 1 < 2$  then the value of  $x$  lies in the interval

- (a)  $(1/2, 3/2)$
- (b)  $(-1/2, 3/2)$
- (c)  $(3/2, 1/2)$
- (d)  $(3/2, -1/2)$

Question 2. If  $x^2 < -4$  then the value of  $x$  is

- (a)  $(-2, 2)$
- (b)  $(2, \infty)$
- (c)  $(-2, \infty)$
- (d) No solution

Question 3. If  $|x| < -5$  then the value of  $x$  lies in the interval

- (a)  $(-\infty, -5)$
- (b)  $(\infty, 5)$
- (c)  $(-5, \infty)$
- (d) No Solution

Question 4. The graph of the inequations  $x \leq 0$ ,  $y \leq 0$ , and  $2x + y + 6 \geq 0$  is

- (a) exterior of a triangle
- (b) a triangular region in the 3rd quadrant
- (c) in the 1st quadrant
- (d) none of these

Question 5. The graph of the inequalities  $x \geq 0$ ,  $y \geq 0$ ,  $2x + y + 6 \leq 0$  is

- (a) a square
- (b) a triangle
- (c)  $\{ \}$
- (d) none of these

Question 6. Solve:  $2x + 1 > 3$

- (a)  $[-1, \infty]$
- (b)  $(1, \infty)$

(c)  $(-\infty, \infty)$

(d)  $(-\infty, 1)$

Question 7. The solution of the inequality  $3(x - 2)/5 \geq 5(2 - x)/3$  is

(a)  $x \in (2, \infty)$

(b)  $x \in [-2, \infty)$

(c)  $x \in [\infty, 2)$

(d)  $x \in [2, \infty)$

Question 8. Solve:  $1 \leq |x - 1| \leq 3$

(a)  $[-2, 0]$

(b)  $[2, 4]$

(c)  $[-2, 0] \cup [2, 4]$

(d) None of these

Question 9. Solve:  $-1/(|x| - 2) \geq 1$  where  $x \in \mathbb{R}$ ,  $x \neq \pm 2$

(a)  $(-2, -1)$

(b)  $(-2, 2)$

(c)  $(-2, -1) \cup (1, 2)$

(d) None of these

Question 10. If  $x^2 < 4$  then the value of  $x$  is

(a)  $(0, 2)$

(b)  $(-2, 2)$

(c)  $(-2, 0)$

(d) None of these

### Very Short:

1. Solve  $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

2. Solve  $3x + 8 > 2$  when  $x$  is a real no.

3. Solve the inequality  $\frac{x}{4} < \frac{(5x-2)(7x-3)}{3 \cdot 5}$

4. If  $4x > -16$  then  $x \square -4$ .

5. Solve the inequality  $\frac{1}{2} \left( \frac{3x}{5} + 4 \right) \geq \frac{1}{3} (x - 6)$

6. Solution set of the in inequations  $2x - 1 \leq 3$  and  $3x + 1 \geq -5$  is.

7. Solve.  $7x + 3 < 5x + 9$ . Show the graph of the solution on number line.
8. Solve the inequality.  $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$
9. Solve  $5x - 3 \leq 3x + 1$  when  $x$  is an integer.
10. Solve  $30x < 200$  when  $x$  is a natural no.

### Short Questions:

1. Solve  $3x - 6 \geq 0$  graphically
2. Ravi obtained 70 and 75 mark in first unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.
3. Ravi obtained 70 and 75 mark in first unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.
4. A company manufactures cassettes and its cost equation for a week is  $C = 300 + 1.5x$  and its revenue equation is  $R = 2x$ , where  $x$  is the no. of cassettes sold in a week. How many cassettes must be sold by the company to get some profit?
5. The longest side of a  $\Delta$  is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the  $\Delta$  is at least 61 cm find the minimum length of the shortest side.

### Long Questions:

1. IQ of a person is given by the formula  $IQ = \frac{MA}{CA} \times 100$   
Where MA is mental age and CA is chronological age. If  $80 \leq IQ \leq 140$  for a group of 12yr old children, find the range of their mental age.
2. Solve graphically  $4x + 3y \leq 60$   $y \geq 2x$   $x \geq 3$   $x, y \geq 0$
3. A manufacturer has 600 liter of a 12% sol. Of acid. How many liters of a 30% acid sol. Must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%.
4. Solve graphically  $x - 2y \leq 3$   $3x + 4y \geq 12$   $x \geq 0$   $y \geq 1$
5. A sol. Of 8% boric acid is to be diluted by adding a 2% boric acid sol. to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 liters of the 8% sol. how many liter of the 2% sol. will have to be added.

**Answer Key:****MCQ:**

1. (b)  $(-1/2, 3/2)$
2. (d) No solution
3. (d) No Solution
4. (b) a triangular region in the 3rd quadrant
5. (c)  $\{ \}$
6. (b)  $(1, \infty)$
7. (d)  $x \in [2, \infty)$
8. (c)  $[-2, 0] \cup [2, 4]$
9. (c)  $(-2, -1) \cup (1, 2)$
10. (b)  $(-2, 2)$

**Very Short Answer:**

1.

$$\frac{3x-4}{2} \geq \frac{x+1}{4} - \frac{1}{1}$$

$$\frac{3x-4}{2} \geq \frac{x+1-4}{4}$$

$$\frac{3x-4}{2} \geq \frac{x-3}{4}$$

$$2(3x-4) \geq (x-3)$$

$$6x-8 \geq x-3$$

$$x \geq 1$$

2.

$$3x+8 > 2$$

$$3x > 2-8$$

$$3x > -6$$

$$x > -2$$

$$(-2, \infty)$$

3.

$$\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$$

$$\frac{x}{4} < \frac{5(5x-2) - 3(7x-3)}{15}$$

$$\frac{x}{4} < \frac{4x-1}{15}$$

$$15x < 16x - 4$$

$$-x < -4$$

$$x > 4$$

$$(4, \infty)$$

4.  $x > -4$ .

5.

$$\frac{1}{2} \left( \frac{3x}{5} + 4 \right) \geq \frac{1}{3} (x - 6)$$

$$\frac{3x}{10} + 2 \geq \frac{x}{3} - 2$$

$$\frac{3x}{10} - \frac{x}{3} \geq -4$$

$$\frac{9x - 10x}{30} \geq -4$$

$$\frac{-x}{30} \geq -4$$

$$-x \geq -120$$

$$x \leq 120$$

$$(-\infty, 120]$$

6.

$$2x - 1 \leq 3, \quad 3x + 1 \geq -5$$

$$\Rightarrow 2x \leq 4, \quad 3x \geq -6$$

$$\Rightarrow x \leq 2, \quad x \geq -2$$

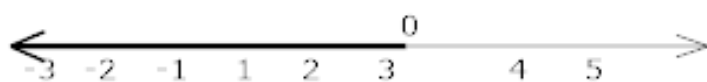
$$\Rightarrow -2 \leq x \leq 2$$

7.

$$7x + 3 < 5x + 9$$

$$2x < 6$$

$$x < 3$$



8.

$$\frac{2x-1}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$$

$$20(2x-1) \geq 3(19x-18)$$

$$40x-20 \geq 57x-54$$

$$-17x \geq -34$$

$$x \leq 2$$

$$(-\infty, 2]$$

9.

$$5x-3 \leq 3x+1$$

$$5x-3x \leq 4$$

$$2x \leq 4$$

$$x \leq 2$$

$$\{\dots, -3, -2, -1, 0, 1, 2\}$$

10.

$$30x < 200$$

$$x < \frac{200}{30}$$

$$x < \frac{20}{3}$$

Solution set of the inequality  $\{1, 2, 3, 4, 5, 6\}$

### Short Answer:

1.

$$3x-6 \geq 0 \dots (i)$$

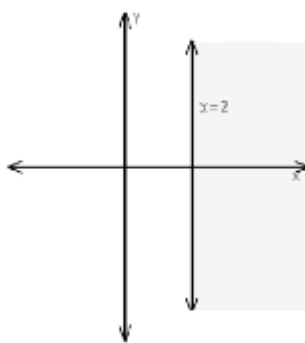
$$3x-6=0$$

$$x=2$$

Put (0,0) in eq. (i)

$$0-6 \geq 0$$

$$0 > 6 \text{ false.}$$



2.

Let Ravi secure  $x$  marks in third test

$$\text{ATQ } \frac{70 + 75 + x}{3} \geq 60$$

$$x \geq 135$$

3.

Let  $x$  and  $x+2$  be consecutive odd natural no.

$$\text{ATQ } x > 10 \dots (i)$$

$$(x) + (x+2) < 40$$

$$x < 19 \dots (ii)$$

From (i) and (ii)

$$(11, 13) \quad (13, 15), \quad (15, 17) \quad (17, 19)$$

4.

Profit = revenue – cost

$$R > C \quad [\text{for to get some profit}]$$

$$2x > 300 + 1.5x$$

$$\frac{1}{2}x > 300$$

$$x > 600$$

5.

Let shortest side be  $x$  cm then the longest side is  $3x$  cm and the third side  $(3x - 2)$  cm.

$$\text{ATQ } (x) + (3x) + (3x - 2) \geq 61$$

$$x \geq 9$$

Length of shortest side is 9 cm.

**Long Answer:**

1.

$$80 \leq IQ \leq 140 \text{ (Given)}$$

$$80 \leq \frac{MA}{CA} \times 100 \leq 140$$

$$80 \leq \frac{MA}{12} \times 100 \leq 140$$

$$80 \times \frac{12}{100} \leq MA \times \frac{100}{12} \times \frac{12}{100} \leq 140 \times \frac{12}{100}$$

$$\frac{96}{10} \leq MA \leq \frac{168}{10}$$

$$9.6 \leq MA \leq 16.8$$

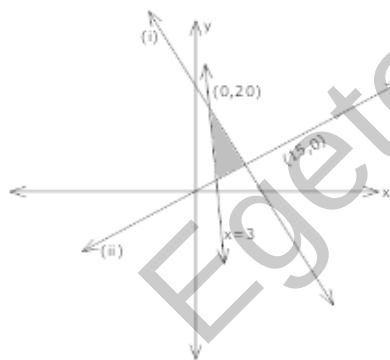
2.  $4x + 3y = 60$ 

|     |          |           |
|-----|----------|-----------|
| $x$ | <b>0</b> | <b>15</b> |
| $y$ | 20       | 0         |

$$y = 2x$$

|     |          |           |
|-----|----------|-----------|
| $x$ | <b>0</b> | <b>20</b> |
| $y$ | 0        | 40        |

$$x = 3$$



3.

Let  $x$  litres of 30% acid sol. Is required to be added.

$$30\%x + 12\% \text{ of } 600 > 15\% \text{ of } (x + 600) \text{ and}$$

$$30\%x + 12\% \text{ of } 600 < 18\% \text{ of } (x + 600)$$

$$\frac{30x}{100} + \frac{12}{100}(600) > \frac{15}{100}(x + 600)$$

$$\frac{30x}{100} + \frac{12}{100}(600) < \frac{18}{100}(x + 600)$$

$$x > 120 \text{ and } x < 300$$

$$\text{i.e. } 120 < x < 300.$$

4.

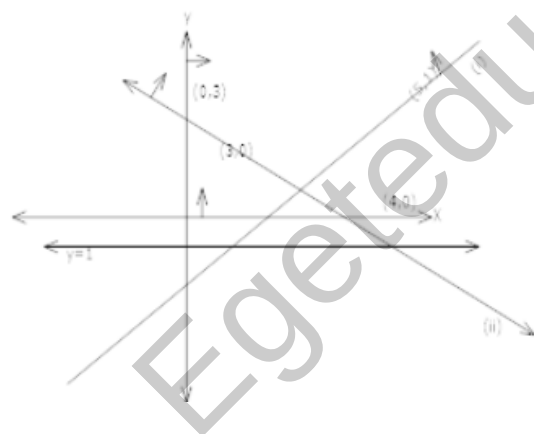
$$x - 2y = 3$$

|     |          |          |
|-----|----------|----------|
| $x$ | <b>3</b> | <b>5</b> |
| $y$ | 0        | 1        |

$$3x + 4y = 12$$

|     |          |          |
|-----|----------|----------|
| $x$ | <b>0</b> | <b>4</b> |
| $y$ | 3        | 0        |

$$y = 1$$



5.

Let  $x$  be added

ATQ 2% of  $x + 8\%$  of 640  $> 4\%$  of  $(640 + x)$

$$\frac{2x}{100} + \frac{8 \times 640}{100} > \frac{4}{100}(640 + x)$$

$$x < 1280 \dots (i)$$

And 12% of  $x + 8\%$  of 640  $< 6\%$  of  $(640 + x)$

$$\frac{2x}{100} + \frac{8 \times 640}{100} < \frac{6}{100}(640 + x)$$

$$x > 320 \dots (ii)$$

From (i) and (ii)

$$320 < x < 1280$$