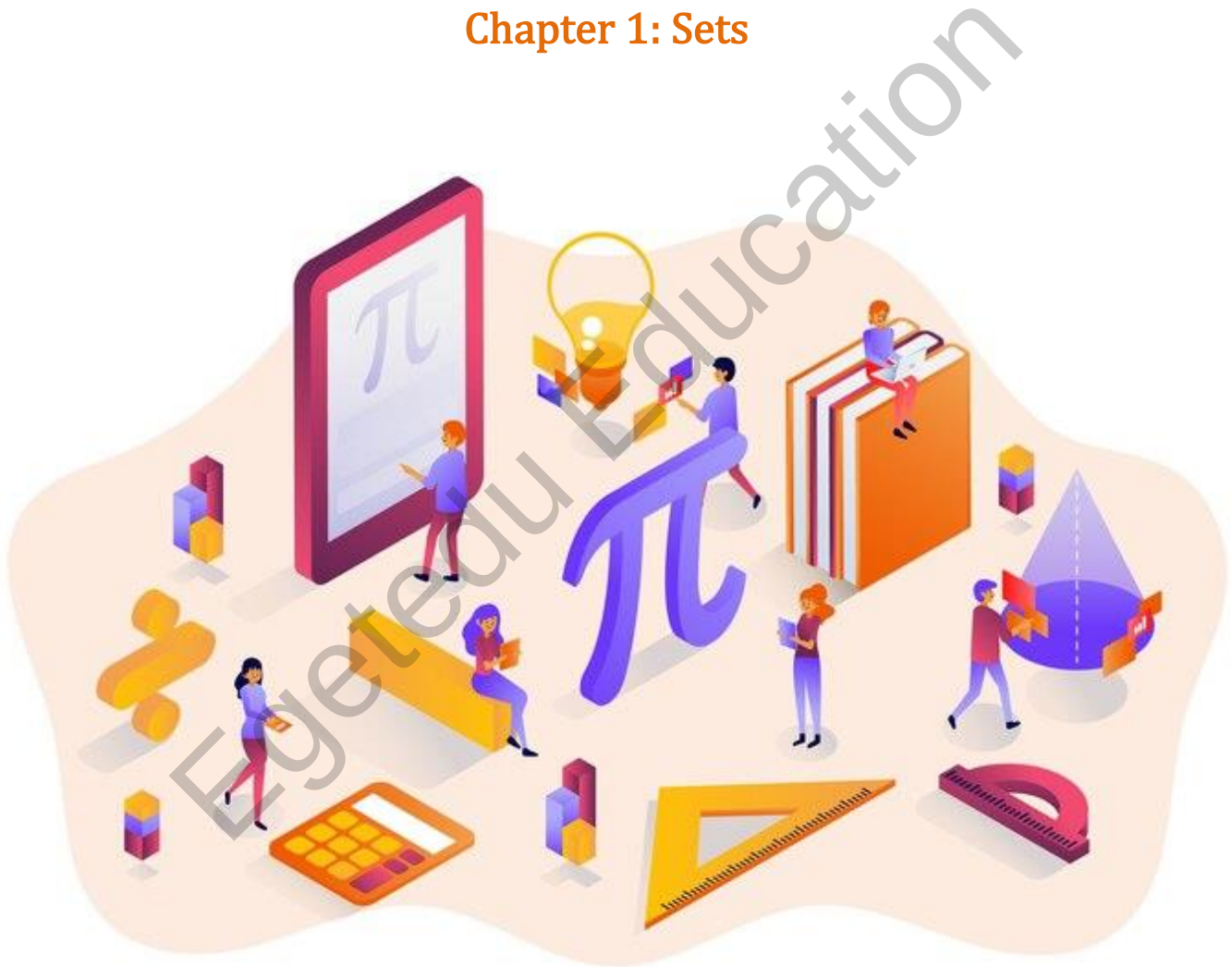


MATHEMATICS

Chapter 1: Sets

Best Self-Learning Online Course



SETS

Key Concepts

1. A set is a well-defined collection of objects.
2. Sets can be represented in two ways—Roster or tabular form and Set builder form.
3. Roster form: All the elements of a set are listed and separated by commas and are enclosed within braces $\{ \}$. Elements are not repeated generally.
4. Set builder form: In set builder form, a set is denoted by stating the properties that its members satisfy.
5. A set does not change if one or more elements of the set are repeated.
6. An empty set is the set having no elements in it. It is denoted by ϕ or $\{ \}$.
7. A set having a single element is called a singleton set.
8. On the basis of number of elements, sets are of two types—Finite and Infinite sets.
9. A finite set is a set in which there are a definite number of elements. Now, ϕ or $\{ \}$ or null set is a finite set as it has 0 number of elements, which is a definite number.
10. A set that is not finite is called an **infinite set**.
11. All infinite sets cannot be described in the roster form.
12. Two sets are equal if they have the exactly same elements.
13. Two sets are said to be equivalent if they have the same **number of elements**.
14. Set A is a subset of set B if every element of A is in B, i.e. there is no element in A which is not in B and is denoted by $A \subset B$.
15. A is a proper subset of B if and only if every element in A is also in B and there exists at least one element in B that is not in A.
16. If A is a proper subset of B, then B is a superset of A and is denoted by $B \supset A$.
17. Let A be a set. Then, the collection of all subsets of A is called the power set of A and is denoted by $P(A)$.

18. Common set notations

N: The set of all natural numbers

Z: The set of all integers

Q: The set of all rational numbers

R: The set of real numbers

Z⁺: The set of positive integers

Q⁺: The set of positive rational numbers

R⁺: The set of positive real numbers

$N \subset R, Q \subset R, Q \not\subset Z, R \not\subset Z$ and $N \subset R^+$

19. Two sets are equal if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

20. Null set ϕ is subset of every set including the null set itself.

21. The set of all the subsets of A is known as the power set of A.

22. **Open interval:** The interval which contains all the elements between a and b excluding a and b. Inset notations:

$$(a, b) = \{x : a < x < b\}$$



Closed interval: The interval which contains all the elements between a and b and also the endpoints a and b is called the **closed interval**.

$$[a, b] = \{x : a \leq x \leq b\}$$

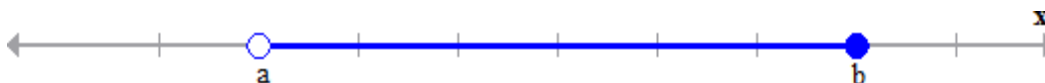


23. Semi-open intervals

$[a, b) = \{x : a \leq x < b\}$ includes all the elements from a to b including a and excluding b



$(a, b] = \{x : a < x \leq b\}$ includes all the elements from a to b excluding a and including b.

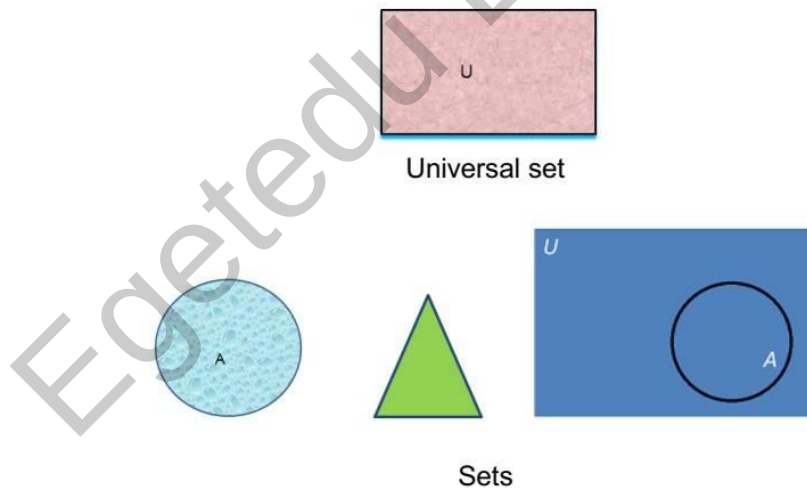


24. Universal set refers to a particular context.

It is the basic set that is relevant to that context. The universal set is usually denoted by U.

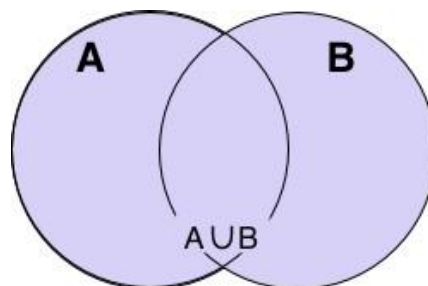
25. Union of sets A and B, denoted by $A \cup B$ is defined as the set of all the elements which are either in A or B or both.

26. Intersection of sets A and B, which are denoted by $A \cap B$, is defined as the set of all the elements, which are common to both A and B.
27. The difference of the sets A and B is the set of elements, which belong to A but not to B and it is written as $A - B$ and read as 'A minus B'.
In set notations $A - B = \{x : x \in A, x \notin B\}$ and $B - A = \{x : x \in B, x \notin A\}$.
28. If the intersection of two non-empty sets is empty, i.e. $A \cap B = \phi$, then A and B are disjoint sets.
29. Let U be the universal set and A be a subset of U. Then the complement of A, written as A' or A^c , is the set of all elements of U that are not in set A.
30. The number of elements present in a set is known as the cardinal number of the set or cardinality of the set. It is denoted by $n(A)$.
31. If A is a subset of U, then A' is also a subset of U.
32. Counting theorems are together known as **Inclusion–Exclusion** Principle. It helps in determining the cardinality of union and intersection of sets.
33. Sets can be represented graphically using venn diagrams. Venn diagrams consist of rectangles and closed curves, usually circles. The universal set is generally represented by a rectangle and its subsets by circles.

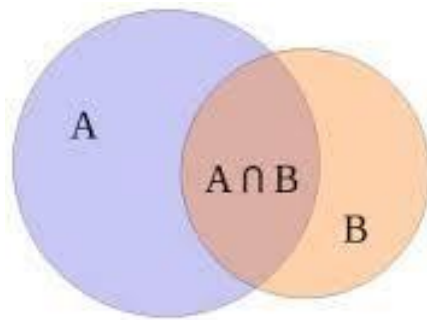


Key Formulae

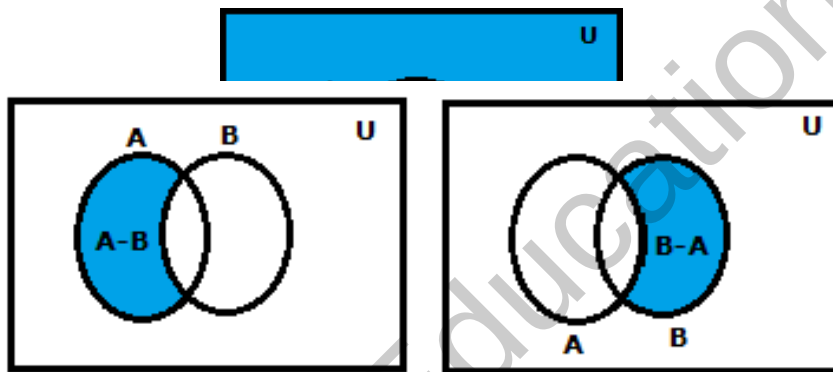
1. Union of sets $A \cup B = \{x : x \in A \text{ or } x \in B\}$



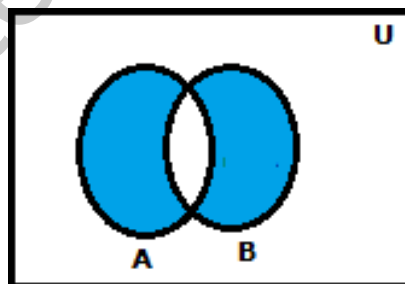
2. Intersection of sets $A \cap B = \{x: x \in A \text{ and } x \in B\}$



3. Complement of a set $A' = \{x: x \in U \text{ and } x \notin A\}$, $A' = U - A$



4. Difference of sets $A - B = \{x: x \in A, x \notin B\}$ and $B - A = \{x: x \in B, x \notin A\}$
5. Symmetric difference of two sets: Let A and B be two sets. Then the symmetric difference of sets $(A - B) \cup (B - A)$ and it is denoted by $A \Delta B$



$$A \Delta B = (A - B) \cup (B - A) = \{x : x \in A, x \in B, x \notin A \cap B\}$$

6. Properties of the operation of union

a. Commutative law:

b. $A \cup B = B \cup A$

c. Associative law:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

d. Law of identity:

e. $A \cup \phi = A$

- f. Idempotent law:
- g. $A \cup A = A$
- h. Law of $U \cup A = U$
- 6. Properties of operation of intersection
 - i) Commutative law:
 $A \cap B = B \cap A$
 - ii) Associative law:
 - iii) $(A \cap B) \cap C = A \cap (B \cap C)$
 - iv) Law of ϕ and U :
 $\phi \cap A = \phi$ and $U \cap A = A$
 - v) Idempotent law:
 $A \cap A = A$
 - vi) Distributive law:
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- 7. Properties of complement of sets
 - a. Complement laws:
 - i. $A \cup A' = U$
 - ii. $A \cap A' = \phi$
 - b. De-Morgan's law:
 - i. $(A \cup B)' = A' \cap B'$
 - ii. $(A \cap B)' = A' \cup B'$
 - iii. Law of double Complementation:
 $(A')' = A$
 - c. Laws of empty set and universal set:
 $\phi' = U$ and $U' = \phi$
- 8. Operations on sets
 - i. $A - B = A \cap B'$
 - ii. $B - A = A' \cap B$
 - iii. $A - B = A \Leftrightarrow A \cap B = \phi$
 - iv. $(A - B) \cup B = A \cup B$
 - v. $(A - B) \cap B = \phi$
 - vi. $A \subseteq B \Leftrightarrow B' \subseteq A'$
 - vii. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
- 9. **Some more important results:** Let A , B and C be three sets. Then, we have
 - i. $A - (B \cap C) = (A - B) \cup (A - C)$

- ii. $A - (B \cup C) = (A - B) \cap (A - C)$
- iii. $A \cap (B - C) = (A \cap B) - (A \cap C)$
- iv. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

10. Counting Theorems

- a. If A and B are finite sets, then the number of elements in the union of two sets is given by $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- b. If A and B are finite sets and $A \cap B = \phi$
then $n(A \cup B) = n(A) + n(B)$
- c. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$
- d. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
- e. Number of elements in exactly two of the sets =
 $n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- f. Number of elements in exactly one of the sets =
 $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$
- g. $n(A' \cup B') = n((A \cap B)') = n(U) - n(A \cap B)$
- h. $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

11. Number of elements in the power set of a set with n elements $= 2^n$.

Number of proper subsets in the power set $= 2^n - 1$.

Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B and written as $A \subset B$ or $B \supset A$ (read as 'A' is contained in 'B' or 'B contains A'). B is called superset of A.
Note:

1. Every set is a subset and superset of itself.
2. If A is not a subset of B, we write $A \not\subset B$.
3. The empty set is the subset of every set.
4. If A is a set with $n(A) = m$, then no. of element A are 2^n and the number of proper subsets of A are $2^n - 1$.

Eg. Let $A = \{3, 4\}$, then subsets of A are $\phi, \{3\}, \{4\}, \{3, 4\}$. Here, $n(A) = 2$ and number of subsets of A = $2^2 = 4$.

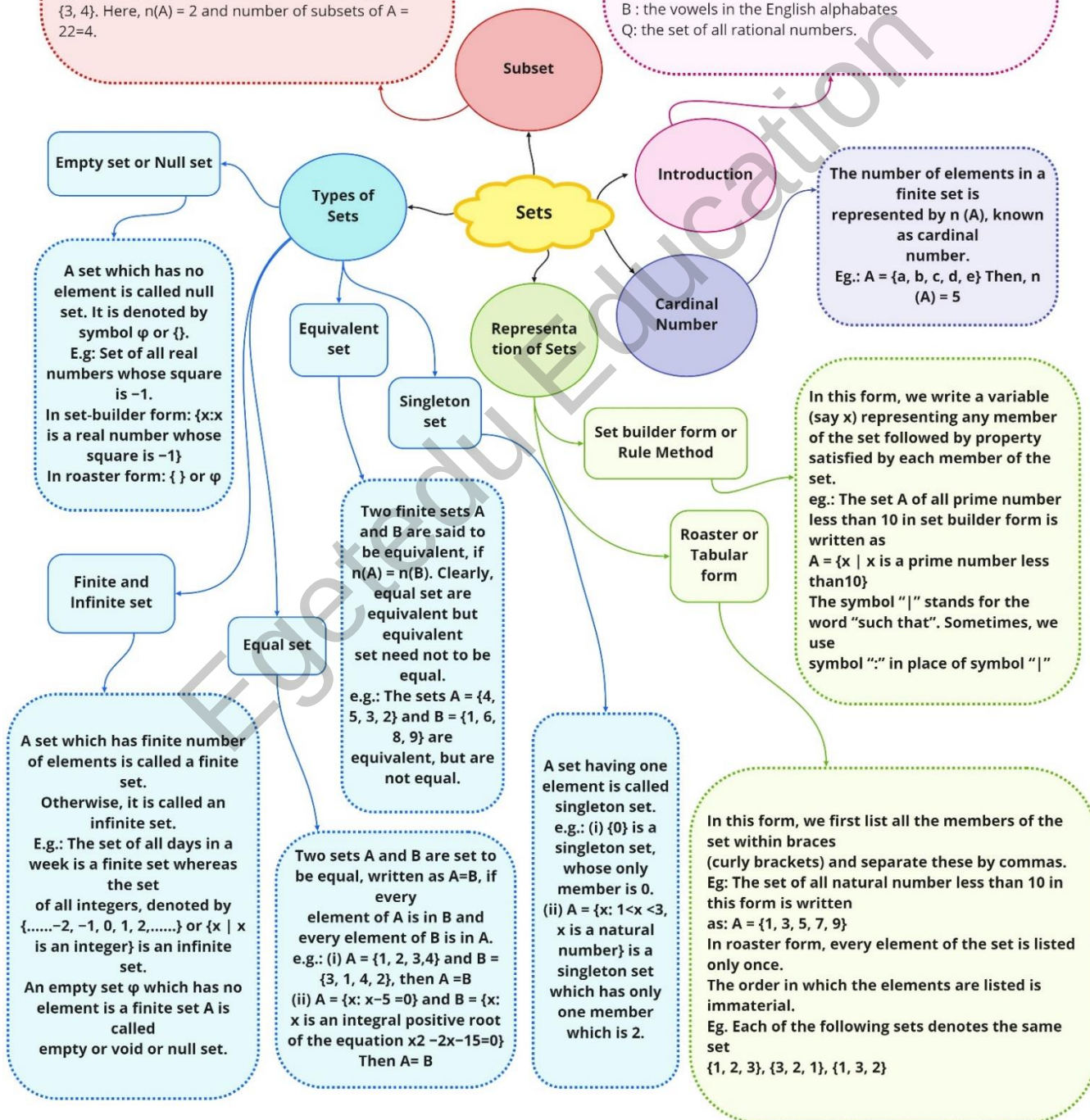
A set is collection of well-defined distinguished objects. The sets are usually denoted by capital letters A, B, C etc., and the members or elements of the set are denoted by lower case letters a, b, c etc.,. If x is a member of the set A, we write $x \in A$ (read as 'x' belongs to A) and if x is not a member of set A, we write $x \notin A$ (read as 'x' doesn't belongs to A). If x and y both belong to A, we write $x, y \in A$.

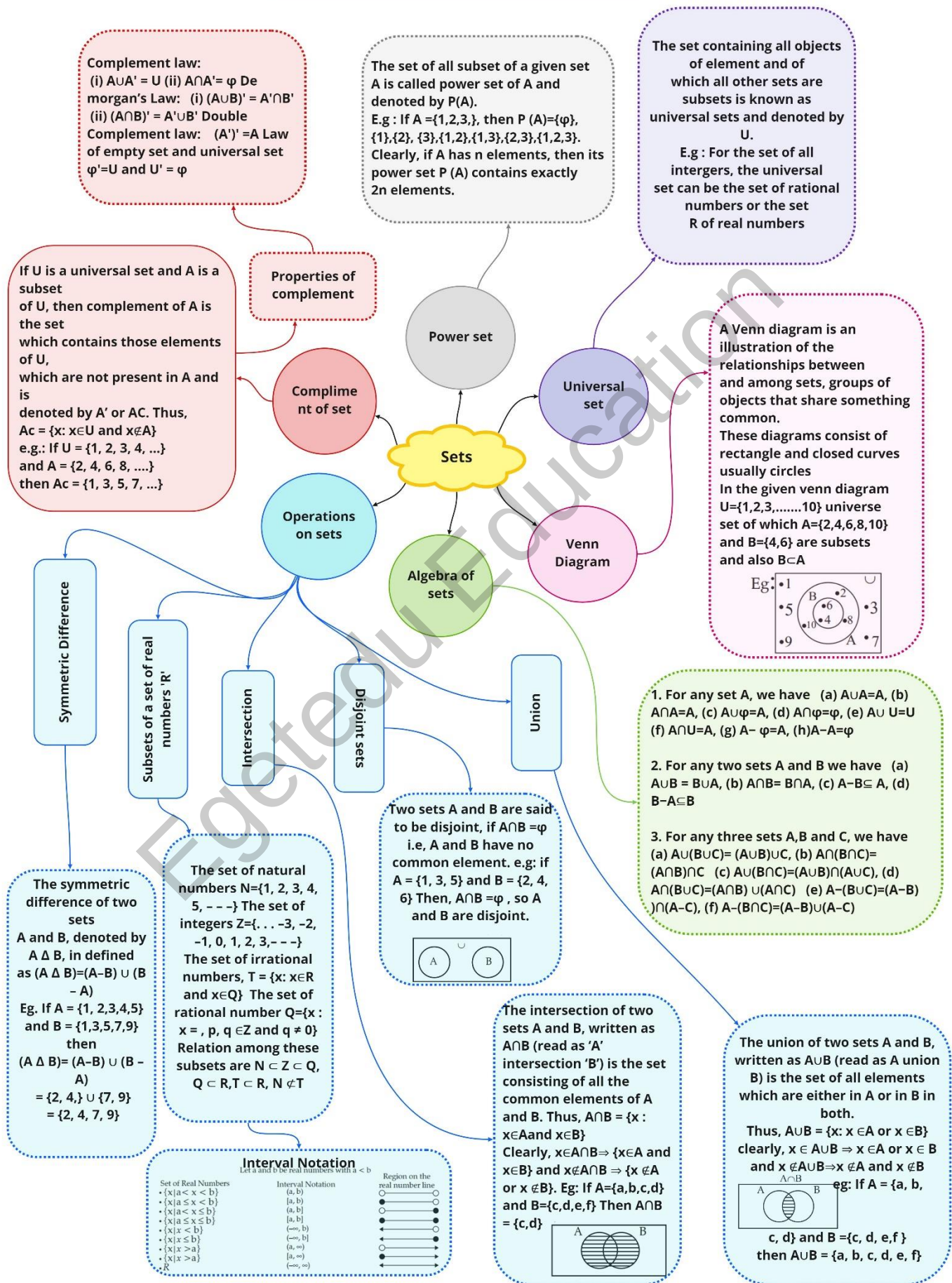
Some examples of sets are: A: odd numbers less than 10

N: the set of all rational numbers

B: the vowels in the English alphabates

Q: the set of all rational numbers.





Important Questions

Multiple Choice questions-

Question 1. If $f(x) = \log \left[\frac{(1+x)}{(1-x)} \right]$, then $f(2x)/(1+x^2)$ is equal to

- (a) $2f(x)$
- (b) $\{f(x)\}^2$
- (c) $\{f(x)\}^3$
- (d) $3f(x)$

Question 2. The smallest set a such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is

- (a) $\{3, 5, 9\}$
- (b) $\{2, 3, 5\}$
- (c) $\{1, 2, 5, 9\}$
- (d) None of these

Question 3. Let $R = \{(x, y) : x, y \text{ belong to } N, 2x + y = 41\}$. The range is of the relation R is

- (a) $\{(2n-1) : n \text{ belongs to } N, 1 \leq n \leq 20\}$
- (b) $\{(2n+2) : n \text{ belongs to } N, 1 < n < 20\}$
- (c) $\{2n : n \text{ belongs to } N, 1 < n < 20\}$
- (d) $\{(2n+1) : n \text{ belongs to } N, 1 \leq n \leq 20\}$

Question 4. Empty set is a?

- (a) Finite Set
- (b) Invalid Set
- (c) None of the above
- (d) Infinite Set

Question 5. Two finite sets have M and N elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of M and N are respectively.

- (a) 6, 3
- (b) 8, 5
- (c) None of these
- (d) 4, 1

Question 6. If the number of elements in a set S are 5. Then the number of

elements of the power set $P(S)$ are?

- (a) 5
- (b) 6
- (c) 16
- (d) 32

Question 7. Every set is a _____ of itself

- (a) None of the above
- (b) Improper subset
- (c) Compliment
- (d) Proper subset

Question 8. If $x \neq 1$, and $f(x) = x + 1 / x - 1$ is a real function, then $f(f(f(2)))$ is

- (a) 2
- (b) 1
- (c) 4
- (d) 3

Question 9. In 3rd Quadrant?

- (a) $X < 0, Y < 0$ (b) $X > 0, Y < 0$
- (c) $X < 0, Y > 0$
- (d) $X < 0, Y > 0$

Question 10. IF $A \cup B = A \cup C$ and $A \cap B = A \cap C$, THEN

- (a) none of these
- (b) $B = C$ only when $A \cap C$
- (c) $B = C$ only when $A \cap B$
- (d) $B = C$

Very Short Questions:

1. The collection of all the months of a year beginning with letter M
2. The collection of difficult topics in Mathematics.

Let $A = \{1, 3, 5, 7, 9\}$. Insert the appropriate symbol $\hat{=}$ or $\hat{\neq}$ in blank spaces: –
(Question- 3,4)

3. $2 \hat{\quad\quad\quad} A$

4. 5 ----- A
5. Write the set $A = \{x: x \text{ is an integer, } -1 \leq x < 4\}$ in roster form
6. List all the elements of the set,
$$A = \{x: x \in \mathbb{Z}, -\frac{1}{2} < x < \frac{11}{2}\}$$
7. Write the set $B = \{3, 9, 27, 81\}$ in set-builder form
8. $A = \{x : x \in \mathbb{N} \text{ and } 3 < x < 4\}$
9. $B = \{x : x \in \mathbb{N} \text{ and } x^2 = x\}$

Short Questions:

1. In a group of 800 people, 500 can speak Hindi and 320 can speak English. Find
 - (i) How many can speak both Hindi and English?
 - (ii) How many can speak Hindi only?
2. A survey shows that 84% of the Indians like grapes, whereas 45% like pineapple. What percentage of Indians like both grapes and pineapple?
3. In a survey of 450 people, it was found that 110 play crickets, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?
4. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?
5. If $A = -3, 5$, $B = (0, 6)$ then find (i) $A - B$, (ii) $A \cup B$
6. In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

Long Questions:

1. In a survey it is found that 21 people like product A, 26 people like product B and 29 like product C. If 14 people like product A and B, 15 people like product B and C, 12 people like product C and A, and 8 people like all the three products. Find
 - (i) How many people are surveyed in all?
 - (ii) How many like product C only?
2. A college awarded 38 medals in football, 15 in basket ball and 20 in cricket. If these medals went to a total of 50 men and only five men got medals in all the

- three sports, how many received medals in exactly two of the three sports?
- There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1 , 50 to chemical C_2 , and 30 to both the chemicals C_1 and C_2 . Find the number of individuals exposed to
 - chemical C_1 but not chemical C_2
 - chemical C_2 but not chemical C_1
 - chemical C_1 or chemical C_2
 - In a survey it was found that 21 peoples liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people like C and A, 15 people like B and C and 8 liked all the three products. Find now many liked product C only
 - A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medal in all the three sports, how many received medals in exactly two of the three sports?

Assertion Reason Questions:

- In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

Assertion (A) : 'The collection of all natural numbers less than 100' is a set.

Reason (R) : A set is a well-defined collection of the distinct objects.

- Both assertion and reason are true and reason is the correct explanation of assertion.
 - Both assertion and reason are true but reason is not the correct explanation of assertion.
 - Assertion is true but reason is false.
 - Assertion is false but reason is true.
- In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

Assertion (A) : The set $D = \{x : x \text{ is a prime number which is a divisor of } 60\}$ in roster form is $\{1, 2, 3, 4, 5\}$.

Reason (R) : The set $E =$ the set of all letters in the word 'TRIGONOMETRY', in the roster form is $\{T, R, I, G, O, N, M, E, Y\}$.

- (i) Both assertion and reason are true and reason is the correct explanation of assertion.
- (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.
- (iii) Assertion is true but reason is false.
- (iv) Assertion is false but reason is true.

Answer Key:

MCQ

1. (a) $2f(x)$
2. (a) $\{3, 5, 9\}$
3. (a) $\{(2n - 1) : n \text{ belongs to } N, 1 \leq n \leq 20\}$
4. (a) Finite Set
5. (a) 6, 3
6. (d) 32
7. (b) Improper subset
8. (d) 3
9. (a) $X < 0, Y < 0$
10. (d) $B = C$

Very Short Answer:

1. Set
2. Not a set
3. $\notin$
4. $\in$
5. $A = \{-1, 0, 1, 2, 3\}$
6. $A = \{0, 1, 2, 3, 4, 5\}$
7. $B = \{x : x = 3n, n \in N \text{ and } 1 \leq n \leq 4\}$
8. Empty set
9. Non-empty set

Short Answer:

Ans: 1. (i) 20 people can speak both Hindi and English

(ii) 480 people can speak Hindi only

Ans: 2. 29% of the Indians like both grapes and pineapple.

Ans: 3. U – set of people surveyed

A – set of people who play cricket

B – set of people who play tennis

Number of people who play neither cricket nor tennis

$$= n(A \cup B)' = n(U) - n(A \cup B)$$

$$= 450 - 200$$

$$= 250$$

Ans: 4. (i) -3, 0; (ii) -3, 6

Ans: 5. Let A denote the set of students taking apple juice and B denote the set of students taking orange juice

$$n(U) = 400, n(A) = 100, n(B) = 150, n(AB) = 75$$

$$n((A' \cap B')) = n(A \cup B)'$$

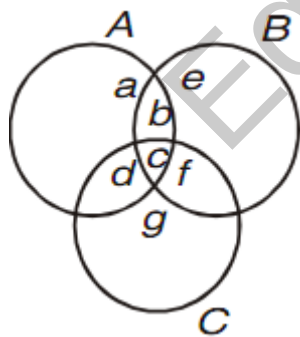
$$= n(U) - n(A \cup B)$$

$$= n(U) - [n(A) + n(B) - n(A \cap B)]$$

$$= 400 - 100 - 150 + 75 = 225$$

Long Answer:

Ans: 1. Let A, B, C denote respectively the set of people who like product A, B, C .

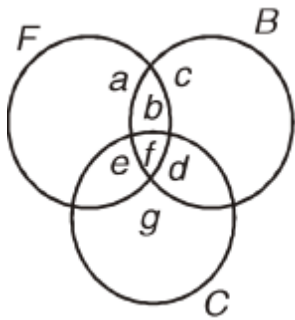


a, b, c, d, e, f, g – Number of elements in bounded region

(i) Total number of Surveyed people = $a + b + c + d + e + f + g = 43$

(ii) Number of people who like product C only = $g = 10$

Ans: 2. people got medals in exactly two of the three sports.



$$f = 5$$

$$a + b + f + e = 38$$

$$b + c + d + f = 15$$

$$e + d + f + g = 20$$

$$a + b + c + d + e + f + g = 50$$

we have to find $b + d + e$

Ans: 3. A denote the set of individuals exposed to the chemical C_1 and B denote the set of individuals exposed to the chemical C_2

$$n(U) = 200, n(A) = 120, n(B) = 50, n(AB) = 30$$

$$(i) n(A-B) = n(A) - n(AB)$$

$$= 120 - 30 = 90$$

$$(ii) n(B-A) = n(B) - n(AB)$$

$$= 50 - 30 = 20$$

$$(iii) n(A \cup B) = n(A) + n(B) - n(AB)$$

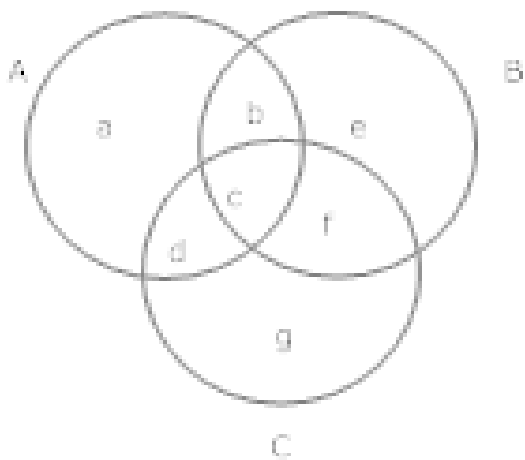
$$= 120 + 50 - 30$$

$$= 140$$

Ans: 4. $a + b + c + d = 21$

$$b + c + e + f = 26$$

$$c + d + f + g = 29$$



$$b + c = 14, c + f = 15, c + d = 12$$

$$c = 8$$

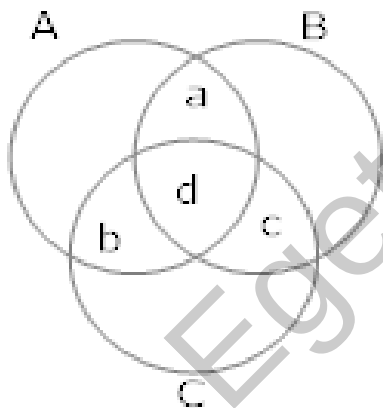
$$d = 4, c = 8, f = 7, b = 6, g = 10, e = 5, a = 3$$

$$\text{like product } c \text{ only} = g = 10$$

Ans: 5. Let A, B and C denotes the set of men who received medals in football, basketball and cricket respectively.

$$n(A) = 38, n(B) = 15, n(C) = 20$$

$$n(A \cup B \cup C) = 58 \text{ and } n(A \cap B \cap C) = 3$$



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$58 = 38 + 15 + 20 - (a + d) - (d + c) - (b + d) + 3$$

$$18 = a + d + c + b + d$$

$$18 = a + b + c + 3d$$

$$18 = a + b + c + 3 \cdot 3$$

$$9 = a + b + c$$

Assertion Reason Answer:

1. (i) Both assertion and reason are true and reason is the correct explanation of assertion.
2. (iv) Assertion is false but reason is true.

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